

# PIM = PARAMETER INDEX MATRIX

Program *MARK* allows the user to build (specify) models using the PIMs. The user can specify a PIM for each set of  $S_j$  and/or  $r_j$ . The PIMs are easy to understand, given knowledge of the parameterization of the  $m_{ij}$  DATA.

For example, using Model  $\{S, r\}$ , the cell probabilities are

$$\begin{array}{ccccc}
 (1-S)r & S(1-S)r & SS(1-S)r & SSS(1-S)r & SSSS(1-S)r \\
 & (1-S)r & S(1-S)r & SS(1-S)r & SSS(1-S)r \\
 & & (1-S)r & S(1-S)r & SS(1-S)r \\
 & & & (1-S)r & S(1-S)r \\
 & & & & (1-S)r
 \end{array}$$

and the corresponding PIMs are

## SURVIVAL PROBABILITIES

1	1	1	1	1
	1	1	1	1
		1	1	1
			1	1
				1

## REPORTING PROBABILITIES

2	2	2	2	2
	2	2	2	2
		2	2	2
			2	2
				2

PIM columns correspond to years and rows correspond to cohorts.

Each parameter gets an index number: 1, 2, 3, . . . for each entry in the PIM. Note, the bold highlighting in the matrix of survival probabilities (above) and the corresponding entries in the PIM (below). This PIM indicates that survival in all years from all cohorts is the same; this is given as parameter #1. Similarly, all the reporting probabilities are constrained to be the same, and denoted as parameter #2.

Using Model  $\{S_t, r\}$  the probabilities are

$$\begin{array}{ccccc}
 (1-S_1)r & S_1(1-S_2)r & S_1S_2(1-S_3)r & S_1S_2S_3(1-S_4)r & S_1S_2S_3S_4(1-S_5)r \\
 & (1-S_2)r & S_2(1-S_3)r & S_2S_3(1-S_4)r & S_2S_3S_4(1-S_5)r \\
 & & (1-S_3)r & S_3(1-S_4)r & S_3S_4(1-S_5)r \\
 & & & (1-S_4)r & S_4(1-S_5)r \\
 & & & & (1-S_5)r
 \end{array}$$

The corresponding PIMs are

## SURVIVAL PROBABILITIES

1	2	3	4	5
	2	3	4	5

## REPORTING PROBABILITIES

6	6	6	6	6
	6	6	6	6

3	4	5	6	6	6
	4	5		6	6
		5			6

In this example (above) the PIM denotes survival probabilities that are year-specific; each year has a different survival probability.

Model  $\{S_t, r_t\}$  in the bass example has  $K = 9$  estimable parameters. Thus, the log-likelihood function is 9 dimensional; each parameter (e.g.,  $r_4$ ) has a dimension. The MLE is that value that of  $r_4$  that maximizes the  $\log_e(\mathcal{L})$ . Actually, the MLEs occur when the function is maximized simultaneously for all 9 parameters. This optimization problem is not one to attempt by hand; however, computer software exists to routinely handle such problems.

Consider the example where tagged juvenile fish are in 3 age-classes and where survival is hypothesized to differ by these 3 classes. Constant sampling probability and no time effects.

Model  $\{S_j, S_s, S_a, r\}$  very different from the models where survival is assumed to be year-dependent. The probabilities are under this age-specific model are

$$\begin{array}{cccccc}
 (1-S_j)r & S_j(1-S_s)r & S_jS_s(1-S_a)r & S_jS_sS_a(1-S_a)r & S_jS_sS_aS_a(1-S_a)r \\
 & (1-S_j)r & S_j(1-S_s)r & S_jS_s(1-S_a)r & S_jS_sS_a(1-S_a)r \\
 & & (1-S_j)r & S_j(1-S_s)r & S_jS_s(1-S_a)r \\
 & & & (1-S_j)r & S_j(1-S_s)r \\
 & & & & (1-S_j)r
 \end{array}$$

( $\neq$  juvenile)

The corresponding PIMs are

SURVIVAL PROBABILITIES				
1	2	3	3	3 ←
	1	2	3	3
		1	2	3
			1	2
				1

REPORTING PROBABILITIES				
4	4	4	4	4
	4	4	4	4
		4	4	4
			4	4
				4