Chapter 3. Models for Birds Banded as Young and Adults

3.1 Introduction

It has often been observed in banding studies that young birds have different survival rates from adults and are more vulnerable to hunters. Thus, the assumption in Chapter 2 that survival and hunting mortality rates are age-independent limits application of the models of Chapter 2 to recoveries from birds banded as adults only. Frequently when banding programs are carried out, both young and adult birds are banded at the same time, and band recoveries are recorded for both age groups. This chapter presents methods for utilizing the records from birds banded as adults and as young to obtain estimates of both adult and first-year survival and recovery rates, and to make tests concerning the age-dependence of these rates.

The Experimental Situation

This situation is basically the same as that of Chapter 2, except that each batch of banded individuals released includes a known number of young birds in addition to the known number of adults. It is important that both young and adults are banded each year or parameters of interest will not be estimable. In particular, banding programs where only young are banded are virtually useless because survival rates of young are not estimable (see Section 3.9). Records of the numbers of birds banded and of the annual band recoveries are kept separately for the two groups of birds. As with the situation discussed in Chapter 2, banding is carried out each year at about the same time for a number of years, and annual recoveries are recorded for each of these years, and possibly for several additional years. Again a "year" of the experiment or banding study is the period between successive bandings, or between anniversaries of the last banding date.

Notation and Definitions

k = number of years at the start of which a release of banded birds is made
ℓ = number of years during which annual band recoveries are recorded, $\ell \geqslant k$,
and we sometimes write $\ell = k + s$, where s = the number of years
when no release is made but recoveries are recorded, $s \geqslant 0$
N_i = number of adults banded in year i, $i = 1, \ldots, k$
M_i = number of young birds banded in year i, $i = 1, \ldots, k$
R_{ij} = number of bands recovered in year j from the *adults* released in
year i, $i = 1, \ldots, k, j = i, \ldots, \ell$
Q_{ij} = number of bands recovered in year j from the *young* birds released
in year i, $i = 1, \ldots, k, j = i, \ldots, \ell$

The data are presented in the form of two tables, the first for recoveries from birds banded as adults and the second for recoveries from birds banded as young. This is illustrated in terms of N_i, M_i, R_{ij}, Q_{ij} for a banding study with $k = 3$, $\ell = 5$, $s = 2$, in Table 3.1 below. Summary totals which are used in the calculation of estimators and tests are indicated in the table and defined below.

Table 3.1. *Symbolic representation of the data for a banding study with $k=3$, $\ell=5$, $(s=2)$, when both young and adult birds are banded and released each year.*

Year banded	Number banded	Year of recovery 1	2	3	4	$\ell=5$	Row totals
		Birds banded as adults					
1	N_1	T_1 R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	$R_{1\cdot}=T_1$
2	N_2		T_2 R_{22}	R_{23}	R_{24}	R_{25}	$R_{2\cdot}$
$k=3$	N_3			T_3 R_{33}	T_4 R_{34}	R_{35}	$R_{3\cdot}$
Column totals		$R_{\cdot 1}$	$R_{\cdot 2}$	$R_{\cdot 3}$	$R_{\cdot 4}$	$R_{\cdot 5}=T_5$	
		Birds banded as young					
1	M_1	U_1 Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	$Q_{1\cdot}=U_1$
2	M_2		U_2 Q_{22}	Q_{23}	Q_{24}	Q_{25}	$Q_{2\cdot}$
3	M_3			U_3 Q_{33}	U_4 Q_{34}	Q_{35}	$Q_{3\cdot}$
Column totals		$Q_{\cdot 1}$	$Q_{\cdot 2}$	$Q_{\cdot 3}$	$Q_{\cdot 4}$	$Q_{\cdot 5}=U_5$	

The array R_{ij} is called the recovery matrix for adults and the array Q_{ij} is called the recovery matrix for young. Row totals, column totals, and certain block totals of these arrays are used in summarizing the data, as indicated in Table 3.1. Thus

$R_{i\cdot}=i^{th}$ row total of the recovery matrix for adults, or equivalently the total number of recoveries from the adults banded in year i, $i=1,\ldots,k$,

$Q_{i\cdot}=i^{th}$ row total of the recovery matrix for adults, or equivalently the total number of recoveries from the young banded in year i, $i=1,\ldots,k$,

$R_{\cdot i}=i^{th}$ column total of the recovery matrix for adults, $i=1,\ldots,\ell$,

$Q_{\cdot i}=i^{th}$ column total of the recovery matrix for young, $i=1,\ldots,\ell$,

$T_1=R_{1\cdot}$

$$T_i=\begin{cases}R_{i\cdot}+T_{i-1}-R_{\cdot i-1} & ,i=2,\ldots,k\\ T_{i-1}-R_{\cdot i-1} & ,i=k+1,\ldots,\ell,\ \text{if } \ell>k\end{cases}$$

$U_1=Q_{1\cdot}$

$$U_i=\begin{cases}Q_{i\cdot}+U_{i-1}-Q_{\cdot i-1} & ,i=2,\ldots,k\\ U_{i-1}-Q_{\cdot i-1} & ,i=k+1,\ldots,\ell\ \text{if } \ell>k.\end{cases}$$

Other useful subtotals which involve elements from both of the recovery matrices are

$W_i=R_{\cdot i}+Q_{\cdot i}-Q_{ii}=$ the total number of recoveries from adults in year i (including recoveries from adult survivors of young banded before year i), $i=1,\ldots,k$,

$T_i+U_i-Q_{i\cdot}=$ the total number of recoveries in years i to ℓ inclusive from all banded adults alive at the start of year i (whether banded as adults or as young), $i=1,\ldots,k$.

Real data for this situation are presented in Table 3.2.

Table 3.2. *Data from a banding study of young and adult male mallards banded preseason in the San Luis Valley, Colorado, 1963-1971 ($k=\ell=9, s=0$).*

Year banded	Number banded	Year of recovery 1963	1964	1965	1966	1967	1968	1969	1970	1971
					Banded as Adults					
1963	231	10	13	6	1	1	3	1	2	0
1964	649		58	21	16	15	13	6	1	1
1965	885			54	39	23	18	11	10	6
1966	590				44	21	22	9	9	3
1967	943					55	39	23	11	12
1968	1,077						66	46	29	18
1969	1,250							101	59	30
1970	938								97	22
1971	312									21
					Banded as Young					
1963	962	83	35	18	16	6	8	5	3	1
1964	702		103	21	13	11	8	6	6	0
1965	1,132			82	36	26	24	15	18	4
1966	1,201				153	39	22	21	16	8
1967	1,199					109	38	31	15	1
1968	1,155						113	64	29	22
1969	1,131							124	45	22
1970	906								95	25
1971	353									38

Subtotals

i	$R_{i\cdot}$	$R_{\cdot i}$	$Q_{i\cdot}$	$Q_{\cdot i}$	T_i	U_i	W_i	$T_i+U_i-Q_{i\cdot}$
1	37	10	175	83	37	175	10	37
2	131	71	168	138	158	260	106	250
3	161	81	205	121	248	327	120	370
4	108	100	259	218	275	465	165	481
5	140	115	194	191	315	441	197	562
6	159	161	228	213	359	478	261	609
7	190	197	191	266	388	456	339	653
8	119	218	120	227	310	310	350	500
9	21	113	38	121	113	121	196	196

Parameters and Assumptions

The parameters and models for the experimental situation of this chapter and that of Chapter 2 are analogous in many ways. Thus, for the reasons given in Chapter 2, the actual harvest rate H and the reporting rate λ are not separately estimable, but their product $H\lambda$ is estimable, and the models are therefore defined in terms of S, the annual survival rate, and $f=H\lambda$, the annual recovery rate. Subscripts on f and S are used to denote year-specificity (i.e., dependence on a particular year), and superscripts (usually primes) on f and S denote age-dependence, or for f, superscripts may indicate a possible dependence of the reporting rate on the year of banding.

As in Chapter 2, assumptions about the specificity of the annual rate parameters can be made progressively more general giving rise to a series of models of increasing complexity. In this chapter, assumptions based on biological and ecological information are made about the parameters H, λ, and S. However, as H and λ are not separately identifiable (estimable), these assumptions must be translated in terms of the parameters f and S into an appropriate model structure. Usually there is not a one-to-one relationship between the model based on the parameters f and S and the original assumptions about H, λ, and S. Because of this a set of assumptions about H, λ, and S is referred to as a hypothesis and denoted H_i, e.g., H_1, and the derived model is referred to as the model under the hypothesis; the model under H_1, or sometimes, the H_1 model. This terminology, which is also used in Chapter 4, has not been made to conform exactly to that of Chapter 2 as it has been used consistently in material of this chapter and the next (cf. Brownie and Robson 1976) and appears in the documentation of related computer output. Specific assumptions, and precise definition of parameters, are described separately for each model in Sections 3.2 to 3.6.

The assumption that banded birds in the population suffer independent fates, on which the development of sampling variance and testing procedures is based, is questionable for species such as geese where young and adults tend to maintain distinct family units for at least part of the first year.

3.2 The Model Under H_1

As in Chapter 2, we begin with one of the most useful models for this situation, which we call the model under H_1. The assumptions of H_1 are:

(1) Annual survival, reporting and harvest rates (hence recovery rates) are year-specific;
(2) annual survival and harvest rates are age-dependent for the first year of life only (i.e., young and adult birds have different survival and harvest rates); and
(3) reporting rates are not dependent on time of release.

In terms of f and S these assumptions lead to defining the following parameters:

f_i' = recovery rate in year i for birds banded and released as young in year i, $i=1,\ldots,k$
S_i' = survival rate for year i for birds banded and released as young in year i, $i=1,\ldots,k-1$ if $\ell=k$, and $1,\ldots,k$ if $\ell>k$
f_i = recovery rate for adults in year i, $i=1,\ldots,\ell$
S_i = survival rate for adults in year i, $i=1,\ldots,\ell-1$.

Tables of expected band recoveries in terms of N_i, M_i, f_i, f_i', S_i and S_i' are used to express the structure of this model. For banded adults, the assumptions of H_1 are the same as those of Model 1 in Chapter 2, so expected recoveries from birds banded as adults in terms of N_i, f_i, S_i are the same under H_1 and Model 1 (see Table 2.3). For birds banded as young, if M_1 are released in the first year, on the average we would expect M_1f_1' bands to be recovered that year, and M_1S_1' of the cohort to survive to adulthood. At the start of the second year M_2 young are banded and released, and there are M_1S_1' survivors (now adults) from the first batch released. Thus, in the second year the expected number of band recoveries from the M_2 new releases is M_2f_2', and the expected number of survivors M_2S_2'. Also in the second year the M_1S_1' survivors of the first batch will reflect the adult rates f_2 and S_2 giving on the average $M_1S_1'f_2$ recoveries and $M_1S_1'S_2$ survivors. Continuing to the third year when M_3 new young are released, we would expect $M_1S_1'S_2f_3$ recoveries from the first batch released, $M_2S_2'f_3$ from the second batch, and M_3f_3' from the new releases. In this way we arrive at the entries in Table 3.3.

Table 3.3 *Expected numbers of band recoveries under H_1 for a banding study with $k=3, \ell=5, s=2$.*

Year banded	Number banded	Year of recovery 1	2	3	4	5
				Birds banded and released as adults		
1	N_1	N_1f_1	$N_1S_1f_2$	$N_1S_1S_2f_3$	$N_1S_1S_2S_3f_4$	$N_1S_1S_2S_3S_4f_5$
2	N_2		N_2f_2	$N_2S_2f_3$	$N_2S_2S_3f_4$	$N_2S_2S_3S_4f_5$
3	N_3			N_3f_3	$N_3S_3f_4$	$N_3S_3S_4f_5$
				Birds banded and released as young		
1	M_1	M_1f_1'	$M_1S_1'f_2$	$M_1S_1'S_2f_3$	$M_1S_1'S_2S_3f_4$	$M_1S_1'S_2S_3S_4f_5$
2	M_2		M_2f_2'	$M_2S_2'f_3$	$M_2S_2'S_3f_4$	$M_2S_2'S_3S_4f_5$
3	M_3			M_3f_3'	$M_3S_3'f_4$	$M_3S_3'S_4f_5$

From Table 3.3 we see that observations corresponding to the off-diagonal elements of the recovery matrix for young (i.e., values of Q_{ij} for $j>i$) will provide information about the adult rate parameters, and any efficient estimation procedure should exploit this fact. We note also that if $\ell>k$, the parameters $S_k,\ldots,S_{\ell-1}$, S_k', $f_{k+1},\ldots,f_\ell$ are not separately estimable, though products such as S_kf_{k+1}, $S_k'f_{k+1}$, etc., are estimable.

Estimation of Parameters

ML estimators of adult and young recovery rates are

$$\hat{f}_i = \frac{R_{i\cdot}}{N_i}\frac{W_i}{T_i+U_i-Q_{i\cdot}} \qquad ,i=1,\ldots,k,$$

$$\hat{f}_i' = \frac{Q_{ii}}{M_i} \qquad ,i=1,\ldots,k,$$

The data in Table 3.2 give, for example,

$$\hat{f}_1 = \frac{R_{1\cdot}}{N_1}\frac{W_1}{T_1+U_1-Q_{1\cdot}} = \frac{37\times 10}{231\times 37} = 0.0433$$

$$\hat{f}_1' = \frac{Q_{11}}{M_1} = \frac{83}{962} = 0.0863$$

$$\hat{f}_2 = \frac{R_{2\cdot}}{N_2}\frac{W_2}{T_2+U_2-Q_{2\cdot}} = \frac{131\times 106}{649\times 250} = 0.0856$$

$$\hat{f}_2' = \frac{Q_{22}}{M_2} = \frac{103}{702} = 0.1467.$$

Bias-adjusted ML estimators of the survival rates are

$$\tilde{S}_i = \frac{R_{i\cdot}}{N_i}\frac{T_i+U_i-Q_{i\cdot}-W_i}{T_i+U_i-Q_{i\cdot}}\frac{N_{i+1}+1}{R_{i+1\cdot}+1} \qquad ,i=1,\ldots,k-1,$$

$$\tilde{S}_i' = \frac{Q_{i\cdot}-Q_{ii}}{M_i}\frac{N_{i+1}+1}{R_{i+1\cdot}+1} \qquad ,i=1,\ldots,k-1.$$

Data in Table 3.2 give

$$\tilde{S}_1 = \frac{R_{1\cdot}}{N_1}\frac{T_1+U_1-Q_{1\cdot}-W_1}{T_1+U_1-Q_{1\cdot}}\frac{N_2+1}{R_{2\cdot}+1} = \frac{37\times(37-10)\times 650}{231\times 37\times 132} = 0.5756$$

$$\tilde{S}_1' = \frac{Q_{1\cdot}-Q_{11}}{M_1}\frac{N_2+1}{R_{2\cdot}+1} = \frac{(175-83)\times 650}{962\times 132} = 0.4709$$

$$\tilde{S}_2 = \frac{131\times(250-106)\times 886}{649\times 250\times 162} = 0.6359$$

$$\tilde{S}_2' = \frac{(168-103)\times 886}{702\times 162} = 0.5064.$$

The FORTRAN program BROWNIE prints the estimates $\hat{f}_i$, $\hat{f}_i'$, $i=1,\ldots,k$, and $\tilde{S}_i$, $\tilde{S}_i'$, $i=1,\ldots,k-1$, as illustrated in Example 3.1a. The unadjusted ML estimators $\hat{S}_i$ and $\hat{S}_i'$, of the adult and young survival rates are computed by the program but are not printed. They are used in the program to obtain the "matrices of expected values" (i.e., the matrices of ML estimates of the expected values) and are defined below.

$$\hat{S}_i = \frac{R_{i\cdot}}{N_i}\frac{T_i+U_i-Q_{i\cdot}-W_i}{T_i+U_i-Q_{i\cdot}}\frac{N_{i+1}}{R_{i+1\cdot}} \qquad ,i=1,\ldots,k-1,$$

$$\hat{S}_i' = \frac{Q_{i\cdot}-Q_{ii}}{M_i}\frac{N_{i+1}}{R_{i+1\cdot}} \qquad ,i=1,\ldots,k-1.$$

The data in Table 3.2 give, for example

$$\hat{S}_1 = \frac{37\times(37-10)\times 649}{231\times 37\times 131} = 0.5791,$$

$$\hat{S}_1' = \frac{(175-83)\times 649}{962\times 131} = 0.4738$$

$$\hat{S}_2 = \frac{131\times(250-106)\times 885}{649\times 250\times 161} = 0.6391$$

$$\hat{S}_2' = \frac{(168-103)\times 885}{702\times 161} = 0.5090\,.$$

Note that these estimates differ only slightly from the bias-adjusted estimates. In general, for sample sizes for which the precision of estimators is good, the difference between $\hat{S}_i$ and $\tilde{S}_i$ is negligible.

Additional Estimates if $\ell > k$

As in Chapter 2, if $\ell > k$, annual rate parameters in years k to ℓ are not separately identifiable, though certain products of these parameters are estimable, e.g., $S_k f_{k+1}$, $S_k S_{k+1} f_{k+2}, \ldots, S_k S_{k+1} \cdots S_{k+s-1} f_{k+s}$. The corresponding ML estimators are

$$\widehat{S_k \cdots S_{k+j-1} f_{k+j}} = \frac{R_{\cdot k+j} + Q_{\cdot k+j}}{T_{k+1} + U_{k+1}} \frac{R_{k\cdot}}{N_k} \left(\frac{T_k + U_k - Q_{k\cdot} - W_k}{T_k + U_k - Q_{k\cdot}} \right) \qquad ,j = 1, \ldots, s,$$

$$\widehat{S_k' f_{k+1}} = \frac{Q_{k\cdot} - Q_{kk}}{M_k} \frac{R_{\cdot k+1} + Q_{\cdot k+1}}{T_{k+1} + U_{k+1}} \qquad ,s = 2$$

and, if $s > 2$,

$$\widehat{S_k' S_{k+1} \cdots S_{k+j-1} f_{k+j}} = \frac{Q_{k\cdot} - Q_{kk}}{M_k} \frac{R_{\cdot k+j} + Q_{\cdot k+j}}{T_{k+1} + U_{k+1}} \qquad ,j = 2, \ldots, s.$$

For the data of Table 3.2, $k = \ell$ and $s = 0$, so the above estimators are not needed.

Sampling Variances, Standard Errors, and Confidence Intervals

Estimates of standard errors, and approximate 95% confidence intervals for f_i, S_i, f_i' and S_i' are contained in the output of the program BROWNIE (see Example 3.1a). The formulae used and some numerical illustrations, again from the data in Table 3.2, are given below.

The notation is similar to that of Chapter 2, thus $\text{var}(\hat{f}_i)$ denotes an estimator of the sampling variance of $\hat{f}_i$, and $\text{se}(\hat{f}_i) = \sqrt{\text{var}(\hat{f}_i)}$ is the corresponding estimator of the standard error of $\hat{f}_i$.

We have

$$var\,(\hat{f}_i) = (\hat{f}_i)^2 \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} + \frac{1}{W_i} - \frac{1}{T_i + U_i - Q_{i\cdot}} \right] \qquad ,i = 1, \ldots, k,$$

$$\text{var}(\tilde{S}_i) = (\tilde{S}_i)^2 \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} + \frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}} + \frac{1}{T_i + U_i - Q_{i\cdot} - W_i} - \frac{1}{T_i + U_i - Q_{i\cdot}} \right] \qquad ,i = 1, \ldots, k-1,$$

$$\text{var}(\hat{f}_i') = \hat{f}_i' \,(1 - \hat{f}_i') / M_i, \qquad ,i = 1, \ldots, k,$$

$$\text{var}(\tilde{S}_i') = (\tilde{S}_i')^2 \left[\frac{1}{Q_{i\cdot} - Q_{ii}} - \frac{1}{M_i} + \frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}} \right] \qquad ,i = 1, \ldots, k-1.$$

Confidence intervals are constructed as in Chapter 2. For example, for the parameter f_i, the approximate 95% confidence interval is given by $(\hat{f}_i - 1.96\ \text{se}(\hat{f}_i), \hat{f}_i + 1.96\ \text{se}(\hat{f}_i))$. This is illustrated for the data of Table 3.2 as follows:

$$\text{var}(\hat{f}_2) = (\hat{f}_2)^2 \left[\frac{1}{R_{2\cdot}} - \frac{1}{N_2} + \frac{1}{W_2} - \frac{1}{T_2 + U_2 - Q_{2\cdot}} \right] = (0.0856)^2 \left[\frac{1}{131} - \frac{1}{649} + \frac{1}{106} - \frac{1}{250} \right] = 0.00008446,$$

$$\text{se}(\hat{f}_2) = \sqrt{0.00008446} = 0.0092,$$

$$1.96 \times \text{se}(\hat{f}_2) = (1.96) \times (0.0092) = 0.0180,$$

and the 95% confidence interval for f_2 is (0.0856 − 0.0180, 0.0856 + 0.0180) or (0.0676, 0.1036). Similarly,

$$\text{var}(\tilde{S}_2) = (0.6359)^2 \left[\frac{1}{131} - \frac{1}{649} + \frac{1}{161} - \frac{1}{885} + \frac{1}{144} - \frac{1}{250} \right] = 0.00570906,$$

$$\text{se}(\tilde{S}_2) = \sqrt{0.00570906} = 0.0756,$$

and

$$1.96 \times \text{se}(\tilde{S}_2) = 0.1482,$$

thus the 95% confidence interval for S_2 is (0.6359 − 0.1482, 0.6359 + 0.1482) or (0.4877, 0.7841). For $\hat{f}_2'$,

$$\text{var}(\hat{f}_2') = \frac{(0.1467) \times (1 - 0.1467)}{702} = 0.00017832,$$

$$\text{se}(\hat{f}_2') = \sqrt{0.00017832} = 0.0134,$$

and, $1.96 \times \text{se}(\hat{f}_2') = 0.0263$, giving the 95% confidence interval for f_2' as (0.1467 − 0.0263, 0.1467 + 0.0263) or (0.1204, 0.1730).

For $\tilde{S}_2'$

$$\text{var}(\tilde{S}_2') = (0.5064)^2 \left[\frac{1}{65} - \frac{1}{702} + \frac{1}{161} - \frac{1}{885}\right] = 0.00488298,$$

$$\text{se}(\tilde{S}_2') = \sqrt{0.00488298} = 0.0699,$$

and $1.96 \times \text{se}(\tilde{S}_2') = 0.1370$, giving the 95% confidence interval for S_2' as (0.5064 − 0.1370, 0.5064 + 0.1370) or (0.3694, 0.6434).

These results can be compared with the computer output in Example 3.1a. Small differences between the calculations above and the computer output are due to the greater degree of accuracy of the latter.

Sampling Covariances and Correlations

Estimates of the sampling covariances between the estimators $\hat{f}_i$, $\tilde{S}_i$, $\hat{f}_i'$ and $\tilde{S}_i'$ are also contained in the output of the program BROWNIE. Estimates are calculated only for those covariances which are not exactly or approximately (for large N_i and M_i) equal to zero. Estimates of corresponding correlations are obtained with these covariance estimates and the variance estimates described above, and are printed out as shown in Example 3.1b. Appropriate formulae and some numerical examples with the data of Table 3.2 are given below.

Let $\text{cov}(\hat{f}_i, \tilde{S}_i)$ be the estimator of the covariance between $\hat{f}_i$ and $\tilde{S}_i$ defined by

$$\text{cov}(\hat{f}_i, \tilde{S}_i) = \hat{f}_i \tilde{S}_i \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} - \frac{1}{T_i + U_i - Q_{i\cdot}}\right] \qquad ,i = 1, \ldots, k-1.$$

An estimate of the correlation between the estimators $\hat{f}_i$ and $\tilde{S}_i$ is

$$\text{corr}(\hat{f}_i, \tilde{S}_i) = \frac{\text{cov}(\hat{f}_i, \tilde{S}_i)}{\text{se}(\hat{f}_i)\ \text{se}(\tilde{S}_i)},$$

where $\text{se}(\hat{f}_i)$ and $\text{se}(\tilde{S}_i)$ are as defined above. Thus,

$$\text{cov}(\hat{f}_2, \tilde{S}_2) = \hat{f}_2 \tilde{S}_2 \left[\frac{1}{R_{2\cdot}} - \frac{1}{N_2} - \frac{1}{T_2 + U_2 - Q_{2\cdot}}\right] = (0.0856)\,(0.6359) \left[\frac{1}{131} - \frac{1}{649} - \frac{1}{250}\right] = 0.0001141,$$

$$\text{corr}(\hat{f}_2, \tilde{S}_2) = \frac{0.0001141}{(0.0092)\,(0.0756)} = 0.1640.$$

Estimates of other covariances and correlations are obtained in a similar way using the following:

$$\text{cov}(\tilde{S}_i, \tilde{S}_{i+1}) = -\tilde{S}_i \tilde{S}_{i+1} \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad ,i = 1, \ldots, k-2,$$

$$\text{cov}(\hat{f}_{i+1}, \tilde{S}_i) = -\hat{f}_{i+1} \tilde{S}_i \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad ,i = 1, \ldots, k-1,$$

$$\text{cov}(\tilde{S}_i, \tilde{S}_i') = \tilde{S}_i \tilde{S}_i' \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad ,i = 1, \ldots, k-1,$$

$$\text{cov}(\hat{f}_i', \tilde{S}_i') = -\frac{\hat{f}_i' \tilde{S}_i'}{M_i} \qquad ,i = 1, \ldots, k-1,$$

$$\text{cov}(\tilde{S}_i', \tilde{S}_{i+1}) = -\tilde{S}_i' \tilde{S}_{i+1} \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad ,i = 1, \ldots, k-2$$

$$\text{cov}(\hat{f}_{i+1}, \tilde{S}_i') = -\tilde{S}_i' \hat{f}_{i+1} \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad ,i = 1, \ldots, k-1.$$

It is emphasized that the above computations provide estimates of the sampling correlations between *estimators*, that is, they reflect a relationship which is a property of the estimators themselves, and which cannot be interpreted as evidence of a relationship between the corresponding parameters. This subject is discussed in more detail in Section 8.4.

Goodness of Fit Test

For the experimental situation of this chapter, we have so far defined a set of assumptions, a corresponding model, and the estimators of parameters of the model. For a given data set, the validity of the estimates obtained will thus depend on whether the assumptions of the model are satisfied. This question cannot usually be answered with complete certainty for a given data set but indications of the adequacy of the model are obtained by means of a goodness of fit test and by tests which compare this model with other models to be defined in Sections 3.3 through 3.7.

A goodness of fit test to H_1 is therefore computed by the program BROWNIE, though not in the conventional way described, for example, in Section 2.2. Under the assumption that the H_1 model holds, the test statistic is approximately distributed as a chi-square variable, and as usual, "large" chi-square values (i.e., values associated with a small probability of occurrence) suggest the model may be incorrect.

The FORTRAN program BROWNIE also provides a means for examining the agreement between the model and the data on the basis of individual observations (i.e., individual R_{ij} or Q_{ij} values), in the following manner. The ML estimate of the expected value under H_1 of each entry in the recovery matrices for adults and young is computed (in terms of the N_i, $\hat{f}_i$, $\hat{S}_i$ for adults, and the M_i, $\hat{f}_i'$ $\hat{S}_i'$, $\hat{f}_i$, $\hat{S}_i$ for young), giving two arrays (cf. Example 3.1c) which are labeled in the output "MATRIX OF EXPECTED VALUES – ADULTS" and "MATRIX OF EXPECTED VALUES – YOUNG." Each observation in the recovery matrices is compared with the estimate of its expected value and the difference, suitably normalized, is approximately a standard normal variable under the assumption that the model holds. The resulting values are printed out in two matrices (one for adults, one for young) each labeled "MATRIX OF STANDARD NORMAL DEVIATES." Inspection of these deviates on an individual basis will help to determine where departures from the model lie, as indicated by unusually small (for instance less than -2), or unusually large (for instance, greater than $+2$), values.

This procedure is illustrated below from the data of Table 3.2. The following notation is needed: Let $\mathbf{E}_{ij}$ represent the ML estimate of the expected value of R_{ij}, and $\mathbf{E}_{ij}'$ represent the ML estimate of the expected value of Q_{ij}; then the corresponding standard normal deviates are

$$Z_{ij} = \frac{R_{ij} - \mathbf{E}_{ij}}{\sqrt{\frac{\mathbf{E}_{ij}}{N_i}(N_i - \mathbf{E}_{ij})}},$$

and

$$Z_{ij}' = \frac{Q_{ij} - \mathbf{E}_{ij}'}{\sqrt{\frac{\mathbf{E}_{ij}'}{M_i}(M_i - \mathbf{E}_{ij}')}}.$$

From Table 3.2, the ij^{th} entry in the recovery matrix for adults, with $i=1$, and $j=2$ is $R_{12}=13$. From Table 3.3 the expected value of R_{12} is $N_1\hat{S}_1\hat{f}_2$ and therefore $\mathbf{E}_{12} = N_1\hat{S}_1\hat{f}_2 = (231)\times(0.5791)\times(0.0856) = 11.45$, giving

$$Z_{12} = \frac{R_{12} - \mathbf{E}_{12}}{\sqrt{\frac{\mathbf{E}_{12}}{N_1}(N_1 - \mathbf{E}_{12})}} = \frac{13 - 11.45}{\sqrt{\frac{11.45}{231}(231 - 11.45)}} = \frac{1.55}{\sqrt{10.8825}} = 0.470$$

which is not an extreme value.

The data value 13, the estimate of its expected value 11.45, and the corresponding normal deviate 0.47, are easily found in the respective matrices for adults, in the computer output of Example 3.1c.

Similarly, from Table 3.2, $Q_{35}=26$, and based on Table 3.3, $\mathbf{E}_{35}' = M_3\hat{S}_3'\hat{S}_4\hat{f}_5 = (1132)\times(0.5936)\times(0.8100)\times(0.0520) = 28.30$, and

$$Z_{35}' = \frac{26 - 28.30}{\sqrt{\frac{28.30}{1132}(1132 - 28.30)}} = -0.44.$$

The difference in the above value of $\mathbf{E}_{35}'$ and the corresponding value in the computer output of Example 3.1c is due to the greater accuracy of the computer calculations. Note that $\mathbf{E}_{ij}$ and $\mathbf{E}_{ij}'$ are obtained with the unadjusted ML estimators $\hat{S}_i$ and $\hat{S}_i'$, rather than the bias-adjusted $\tilde{S}_i$ and $\tilde{S}_i'$.

Examination of the deviates in the matrices for adults and young in the output of Example 3.1c shows that there are a few extreme values but on the whole, the agreement between the model and the data seems good.

An Example

Output from the computer analysis of the mallard data of Table 3.2 appears in Example 3.1. The output is on the whole self-explanatory. For obvious reasons N_i appears as N(I), M_i as M(I), f_i and $\hat{f}_i$ as F(I), S_i and $\tilde{S}_i$ as S(I), cov($\hat{f}_i$, $\tilde{S}_i$) as COVAR(F(I),S(I)), corr($\hat{f}_i$, $\tilde{S}_i$) as CORR(F(I),S(I)), and so on. In this example we note that in any year, the estimate of the recovery rate for young is *higher* than the estimate of the adult recovery rate, and the estimate of survival for young is *lower* than the estimate of adult survival.

The quantity $\bar{F}=0.0668$, which appears (Example 3.6a) below the column of estimates F(I) $\equiv \hat{f}_i$, is simply the average of the $\hat{f}_i$. It is an estimate of the average adult recovery rate during 1963-71. Similarly $\bar{F}'=0.1049$, $\bar{S}=0.6248$ and $\bar{S}'=0.5179$ are averages of the appropriate year-specific estimates, and are estimates of the corresponding average recovery and survival rates. Confidence intervals based on these average estimates are smaller than those based on individual (annual) estimates and $\bar{S}$ and $\bar{S}'$, in particular, may provide useful information in data sets where N_i and M_i are not large.

Examination of the matrices of standard normal deviates suggests that the agreement between model and data is reasonable. There are no really extreme entries in the matrices, and no obvious trends to suggest departure from the model.

Tests related to the model under H_1 appear in another part of the output as can be seen in the complete output for the same data in Example 3.5 at the end of this chapter.

3.3 The Model Under H_{02}

We now define another useful model for the two-age-class situation. The assumptions on which this model is based are collectively referred to as the hypothesis H_{02}. This hypothesis is more restrictive than H_1 in that survival rates are assumed to be constant from year to year, but is otherwise the same as H_1.

The assumptions of H_{02} are:

(1) Annual reporting and harvest rates (and hence recovery rates) are year-specific;
(2) annual survival and harvest rates are age-dependent for the first year of life only; and
(3) annual survival rates are otherwise constant from year to year.

The parameters of the model H_{02} are:

f_i' = recovery rate in year i for birds banded and released as young in year i, $i=1,\ldots,k$,
S' = constant annual survival rate for young,
f_i = recovery rate for adults in year i, $i=1,\ldots,\ell$,
S = constant annual survival rate for adults.

As before, tables of expected numbers of band returns, in terms of N_i, M_i, f_i, f_i', S, and S' are used to express the structure of this model.

Table 3.4. *Expected numbers of band recoveries under H_{02} for a banding study with $k=3$, $\ell=5$, $s=0$.*

Year banded	Number banded	Year of recovery 1	2	3	4	5
		Birds banded and released as adults				
1	N_1	N_1f_1	N_1Sf_2	N_1SSf_3	N_1SSSf_4	N_1SSSSf_5
2	N_2		N_2f_2	N_2Sf_3	N_2SSf_4	N_2SSSf_5
3	N_3			N_3f_3	N_3Sf_4	N_3SSf_5
		Birds banded and released as young				
1	M_1	M_1f_1'	$M_1S'f_2$	$M_1S'Sf_3$	$M_1S'SSf_4$	$M_1S'SSSf_5$
2	M_2		M_2f_2'	$M_2S'f_3$	$M_2S'Sf_4$	$M_2S'SSf_5$
3	M_3			M_3f_3'	$M_3S'f_4$	$M_3S'Sf_5$

Example 3.1a

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

THE HYPOTHESIS H1. (SEE BROWNIE AND ROBSON, 1974. CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

ASSUMPTIONS: (1) ANNUAL SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC.

(2) YOUNG BIRDS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES FROM THOSE OF ADULTS.

PARAMETERS:

F(I) = BAND RECOVERY RATE FOR ADULTS IN YEAR I.
S(I) = SURVIVAL RATE FOR ADULTS IN YEAR I.
F'(I) = BAND RECOVERY RATE FOR YOUNG IN YEAR I.
S'(I) = SURVIVAL RATE FOR YOUNG IN YEAR I.

STRUCTURE OF THE MODEL UNDER H1 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS

N(1)F(1)	N(1)S(1)F(2)	N(1)S(1)S(2)F(3)	N(1)S(1)S(2)S(3)F(4)
	N(2)F(2)	N(2)S(2)F(3)	N(2)S(2)S(3)F(4)
		N(3)F(3)	N(3)S(3)F(4)

BANDED AS YOUNG

M(1)F'(1)	M(1)S'(1)F(2)	M(1)S'(1)S(2)F(3)	M(1)S'(1)S(2)S(3)F(4)
	M(2)F'(2)	M(2)S'(2)F(3)	M(2)S'(2)S(3)F(4
		M(3)F'(3)	M(3)S'(3)F(4)

ESTIMATES UNDER H1

I	YR	F(I) ESTIMATE	F(I) STANDARD ERROR	F(I) 95% CONFIDENCE INTERVAL	S(I) ESTIMATE	S(I) STANDARD ERROR	S(I) 95% CONFIDENCE INTERVAL
1	1963	0.0433	0.0134	0.0170 - 0.0695	0.5756	0.1134	0.3533 - 0.7978
2	1964	0.0856	0.0092	0.0676 - 0.1036	0.6359	0.0756	0.4878 - 0.7840
3	1965	0.0590	0.0061	0.0470 - 0.0710	0.6665	0.0787	0.5122 - 0.8207
4	1966	0.0628	0.0067	0.0496 - 0.0760	0.8051	0.0977	0.6136 - 0.9967
5	1967	0.0520	0.0050	0.0422 - 0.0619	0.6496	0.0724	0.5078 - 0.7914
6	1968	0.0633	0.0055	0.0525 - 0.0740	0.5525	0.0581	0.4387 - 0.6664
7	1969	0.0789	0.0061	0.0670 - 0.0908	0.5719	0.0663	0.4419 - 0.7020
8	1970	0.0888	0.0080	0.0730 - 0.1046	0.5415	0.1286	0.2894 - 0.7936
9	1971	0.0673	0.0142	0.0395 - 0.0951			

	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$	= 0.0668	0.0029	0.0610 - 0.0726	$\bar{S}$	= 0.6248	0.0214	0.5828 - 0.6668

I	YR	F'(I) ESTIMATE	F'(I) STANDARD ERROR	F'(I) 95% CONFIDENCE INTERVAL	S'(I) ESTIMATE	S'(I) STANDARD ERROR	S'(I) 95% CONFIDENCE INTERVAL
1	1963	0.0863	0.0091	0.0685 - 0.1040	0.4709	0.0594	0.3545 - 0.5874
2	1964	0.1467	0.0134	0.1205 - 0.1729	0.5064	0.0699	0.3694 - 0.6434
3	1965	0.0724	0.0077	0.0573 - 0.0875	0.5891	0.0717	0.4486 - 0.7297
4	1966	0.1274	0.0096	0.1085 - 0.1463	0.5909	0.0716	0.4506 - 0.7312
5	1967	0.0909	0.0083	0.0746 - 0.1072	0.4776	0.0610	0.3581 - 0.5971
6	1968	0.0978	0.0087	0.0807 - 0.1150	0.6521	0.0723	0.5104 - 0.7939
7	1969	0.1096	0.0093	0.0914 - 0.1278	0.4635	0.0678	0.3307 - 0.5964
8	1970	0.1049	0.0102	0.0849 - 0.1248	0.3926	0.1133	0.1705 - 0.6147
9	1971	C.1076	0.0165	0.0753 - 0.1400			

	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$	= 0.1049	0.0035	0.0979 - 0.1118	$\bar{S}'$	= 0.5179	0.0265	0.4659 - 0.5699

Example 3.1b

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H1

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1963	-0.000107861	-0.071052198	-0.002229831	-0.260315312
2	1964	0.000113888	0.164048642	-0.002153369	-0.362159372
3	1965	0.000093530	0.194625663	-0.004058998	-0.527787699
4	1966	0.000277319	0.420584456	-0.003181363	-0.449952175
5	1967	0.000145477	0.398951049	-0.001924280	-0.457790708
6	1968	0.000130008	0.407034557	-0.001410437	-0.365958111
7	1969	0.000132313	0.329578633	-0.002272297	-0.266319993
8	1970	0.000256653	0.248228325		

I	YR	COVAR(F(I+1),S(I))	CORR(F(I+1),S(I))	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))
1	1963	-0.000300121	-0.288098030	-0.000042236	-0.078513900
2	1964	-0.000190633	-0.413183861	-0.000105842	-0.113419513
3	1965	-0.000316565	-0.596183595	-0.000037700	-0.068248954
4	1966	-0.000254852	-0.517437047	-0.000062679	-0.090986152
5	1967	-0.000220346	-0.553958592	-0.000036215	-0.071550000
6	1968	-0.000194598	-0.553483970	-0.000055240	-0.087394643
7	1969	-0.000372666	-0.698801335	-0.000044936	-0.071363550
8	1970	-0.001618715	-0.887260327	-0.000045436	-0.039394739

I	YR	COVAR(S'(I),S(I))	CORR(S'(I),S(I))	COVAR(S'(I),S(I+1))	CORR(S'(I),S(I+1))
1	1963	0.001651412	0.245122223	-0.001824454	-0.406359058
2	1964	0.001636182	0.309903856	-0.001714925	-0.311848868
3	1965	0.002970106	0.526383420	-0.003588038	-0.512085954
4	1966	0.002893728	0.413544294	-0.002334868	-0.450695864
5	1967	0.001663408	0.377116138	-0.001414800	-0.399447721
6	1968	0.001608233	0.382845799	-0.001664660	-0.347049369
7	1969	0.001945246	0.432637488	-0.001841692	-0.211269655
8	1970	0.009441458	0.647830523		

I	YR	COVAR(S'(I),F(I+1))	CORR(S'(I),F(I+1))
1	1963	-0.000245560	-0.449728519
2	1964	-0.000151819	-0.355785182
3	1965	-0.000279834	-0.578446973
4	1966	-0.000187041	-0.518292501
5	1967	-0.000162006	-0.483359803
6	1968	-0.000229674	-0.524885804
7	1969	-0.000302045	-0.554353714
8	1970	-0.001173591	-0.730147293

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

COVAR($\bar{S}$,$\bar{F}$) = -0.000033711 CORR($\bar{S}$,$\bar{F}$) = -0.533301536

COVAR($\bar{S}'$,$\bar{F}'$)= -0.000005976 CORR($\bar{S}'$,$\bar{F}'$)= -0.063481008

Example 3.1c

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

MATRIX OF DATA VALUES -- ADULTS H1

1	10.00	13.00	6.00	1.00	1.00	3.00	1.00	2.00	0.0
2	0.0	58.00	21.00	16.00	15.00	13.00	6.00	1.00	1.00
3	0.0	0.0	54.00	39.00	23.00	18.00	11.00	10.00	6.00
4	0.0	0.0	0.0	44.00	21.00	22.00	9.00	9.00	3.00
5	0.0	0.0	0.0	0.0	55.00	39.00	23.00	11.00	12.00
6	0.0	0.0	0.0	0.0	0.0	66.00	46.00	29.00	18.00
7	0.0	0.0	0.0	0.0	0.0	0.0	101.00	59.00	30.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	97.00	22.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.00

MATRIX OF EXPECTED VALUES -- ADULTS

1	10.00	11.45	5.04	3.60	2.42	1.92	1.33	0.86	0.37
2	0.0	55.54	24.47	17.49	11.74	9.32	6.45	4.18	1.79
3	0.0	0.0	52.22	37.32	25.05	19.89	13.77	8.93	3.83
4	0.0	0.0	0.0	37.05	24.87	19.75	13.67	8.86	3.80
5	0.0	0.0	0.0	0.0	49.07	38.97	26.97	17.49	7.50
6	0.0	0.0	0.0	0.0	0.0	68.14	47.17	30.58	13.11
7	0.0	0.0	0.0	0.0	0.0	0.0	98.64	63.95	27.41
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	83.30	35.70
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.00

MATRIX OF STANDARD NORMAL DEVIATES -- ADULTS

1	0.00	0.47	0.43	-1.38	-0.92	0.78	-0.29	1.23	-0.61
2	0.0	0.34	-0.72	-0.36	0.96	1.21	-0.18	-1.56	-0.59
3	0.0	0.0	0.25	0.28	-0.42	-0.43	-0.75	0.36	1.11
4	0.0	0.0	0.0	1.18	-0.79	0.52	-1.28	0.05	-0.41
5	0.0	0.0	0.0	0.0	0.87	0.01	-0.78	-1.57	1.65
6	0.0	0.0	0.0	0.0	0.0	-0.27	-0.17	-0.29	1.36
7	0.0	0.0	0.0	0.0	0.0	0.0	0.25	-0.64	0.50
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.57	-2.34
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00

MATRIX OF DATA VALUES -- YOUNG H1

1	83.00	35.00	18.00	16.00	6.00	8.00	5.00	3.00	1.00
2	0.0	103.00	21.00	13.00	11.00	8.00	6.00	6.00	0.0
3	0.0	0.0	82.00	36.00	26.00	24.00	15.00	18.00	4.00
4	0.0	0.0	0.0	153.00	39.00	22.00	21.00	16.00	8.00
5	0.0	0.0	0.0	0.0	109.00	38.00	31.00	15.00	1.00
6	0.0	0.0	0.0	0.0	0.0	113.00	64.00	29.00	22.00
7	0.0	0.0	0.0	0.0	0.0	0.0	124.00	45.00	22.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	95.00	25.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.00

MATRIX OF EXPECTED VALLES -- YOUNG

1	83.00	39.01	17.19	12.28	8.25	6.55	4.53	2.94	1.26
2	0.0	103.00	21.08	15.07	10.11	8.03	5.56	3.60	1.54
3	0.0	0.0	82.00	42.19	28.33	22.49	15.57	10.09	4.33
4	0.0	0.0	0.0	153.00	37.16	29.50	20.42	13.24	5.67
5	0.0	0.0	0.0	0.0	109.00	36.43	25.22	16.35	7.01
6	0.0	0.0	0.0	0.0	0.0	113.00	59.70	38.71	16.59
7	0.0	0.0	0.0	0.0	0.0	0.0	124.00	46.90	20.10
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	95.00	25.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.00

MATRIX OF STANDARD NORMAL DEVIATES -- YOUNG

1	0.0	-0.66	0.20	1.07	-0.79	0.57	0.22	0.04	-0.23
2	0.0	0.0	-0.02	-0.54	0.28	-0.01	0.19	1.27	-1.24
3	0.0	0.0	0.0	-0.97	-0.44	0.32	-0.15	2.50	-0.16
4	0.0	0.0	0.0	0.0	0.31	-1.40	0.13	0.76	0.98
5	0.0	0.0	0.0	0.0	0.0	0.26	1.16	-0.34	-2.28
6	0.0	0.0	0.0	0.0	0.0	0.0	0.57	-1.59	1.34
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.28	0.43
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

It is readily seen, for example by comparing Table 3.4 with Table 3.3, that there are fewer parameters to be estimated under H_{02} than under H_1. All the parameters of interest under H_{02}, that is $f_1, f_2, \ldots, f_\ell, f_1', f_2', \ldots, f_k'$, S and S', are identifiable, and hence, estimable.

Before discussing estimation, we note that there is a relationship between the assumptions of H_{02} and the hypothesis of compensatory natural mortality, in that harvest or exploitation rates are assumed to vary from year to year, but total mortality (the complement of S) is assumed to be constant. However, the resulting model cannot be unambiguously associated with compensatory mortality because of the nonidentifiability of the actual harvest rate H. Thus variable f_i and constant S arise if H and S are both constant, but λ_i (the reporting rate) varies. The assumption of a constant exploitation rate is usually known to be unreasonable, however. Testing the validity of the H_{02} model, and in particular comparing H_{02} with H_1 are therefore of interest (see Section 3.7).

Estimation of Parameters

The ML estimators of the parameters cannot be expressed explicitly as simple functions of the data in contrast to the ML estimators under H_1. Program BROWNIE contains an algorithm for obtaining these estimates which are printed out together with estimates of their standard errors and 95% confidence intervals. More precisely, the likelihood equations cannot be solved explicitly for $\hat{f}_i$, $\hat{f}_i'$, $\hat{S}$, and $\hat{S}'$, and solutions must be obtained by numerical methods. The method of scoring is used and is described in more detail in Appendix B and in Seber (1973:16-18).

Estimates of certain sampling covariances and correlations between the estimators (those most likely to be of interest) are also printed. Estimation under this model is illustrated in the computer output in Example 3.2. Note that if $\ell > k$ $(s > 0)$, although the parameters $f_{k+1}, f_{k+2}, \ldots, f_{k+s}$ are each estimable, their estimates are not printed out. Also, only the first k estimates $\hat{f}_i$ are averaged to obtain $\hat{\bar{f}}$, an estimate of the average adult recovery rate in years 1 to k. This is because for data sets with typical values of N_i and M_i (i.e., values less than 500) the variances of the estimators $\hat{f}_{k+1}, \ldots, \hat{f}_{k+s}$ and corresponding confidence intervals tend to be so large that the estimates are of little use.

Goodness of Fit Test

A goodness of fit test to the model under H_{02} is computed in the conventional way described in Section 2.2. The resulting chi-square value and associated degrees of freedom are printed, and as usual, large chi-square values indicate that some of the assumptions of the model may be violated.

If $\mathbf{E}_{ij}$ and $\mathbf{E}_{ij}'$ represent the ML estimates of the expected values of R_{ij} and Q_{ij}, respectively, then the goodness of fit chi-square statistic is

$$\sum_{i=1}^{k} \sum_{j=i}^{\ell} \left\{ \frac{(R_{ij} - \mathbf{E}_{ij})^2}{\mathbf{E}_{ij}} + \frac{(Q_{ij} - \mathbf{E}_{ij}')^2}{\mathbf{E}_{ij}'} \right\} + \sum_{i=1}^{k} \left\{ \frac{[(N_i - R_{i\cdot}) - (N_i - \mathbf{E}_{i\cdot})]^2}{N_i - \mathbf{E}_{i\cdot}} + \frac{[(M_i - Q_{i\cdot}) - (M_i - \mathbf{E}_{i\cdot}')]^2}{M_i - \mathbf{E}_{i\cdot}'} \right\},$$

where $N_i - R_{i\cdot}$ and $M_i - Q_{i\cdot}$ are the number of bands never recovered from the i^{th} releases of adults and young, respectively, and $N_i - \mathbf{E}_{i\cdot}$ and $M_i - \mathbf{E}_{i\cdot}'$ are the ML estimates of the corresponding expected numbers. The degrees of freedom are $k^2 + 2ks - k - s - 2$.

In practice, some of the $\mathbf{E}_{ij}$ and $\mathbf{E}_{ij}'$ values will be too small to justify the chi-square approximation, and some pooling is necessary. As necessary, program BROWNIE will pool expected recoveries within a row from right to left in the matrix until a combined value of at least 2 is obtained. At the same time, corresponding R_{ij}'s or Q_{ij}'s are pooled also. A degree of freedom is lost for each R_{ij} or Q_{ij} pooled.

Again, a rough idea of the agreement between model and data can be obtained by comparing individual data values with their estimated expected values.

Thus, as under H_1, corresponding to each of the recovery matrices, a matrix of "expected values," and a matrix of "standard normal deviates" are printed.

An Example

The analysis described above for the model under H_{02} is illustrated for data from a study on male blue-winged teal banded preseason in Saskatchewan during 1962-68, with recoveries recorded through the 1973 season. For these data, $k = 7$, $\ell = 12$, $s = 5$, and the data with basic subtotals appear in the form of sample output from program BROWNIE in Example 3.2a.

For reasons given above, although estimates of each of the annual recovery rates $f_1, f_2, \ldots, f_{12}$ are computed, only the first $k(=7)$ are printed (see Example 3.2a). Note that estimates of recovery rates for young are higher than those for adults, and the estimate of the constant survival rate for young is lower than that for adults, indicating there is age-dependence of these parameters.

The estimates of both young and adult recovery rates are low compared with estimates from the mallard data used for Examples 3.1 and 3.5. For example, for the blue-winged teal, averaging the H_{02} estimates of adult recovery gives $\hat{\bar{f}}=0.0067$, while for the mallard data of Example 3.5c, $\hat{\bar{f}}=0.0646$.

These low recovery rates are reflected in the low precision of $\hat{S}$, and $\hat{S}'$ (and hence in the large confidence intervals for S, and S'), even though the number of birds banded in most years is over 2,000. Thus comparison of Example 3.2a with Example 3.5c shows that for the blue-winged teal, $\hat{S}=0.6356$ with standard error 0.0240 and for the mallards, $\hat{S}=0.6515$ with standard error 0.0120, although the total number of releases for the blue-winged teal is considerably greater than that for the mallards. This should indicate that the number of birds which must be banded to obtain a given level of precision for the survival estimates will depend greatly on the effective recovery rate for the species and flyway of interest. This subject is discussed further in Section 9.3.

Low recovery rates are also reflected in the "MATRICES OF EXPECTED VALUES," the entries of which are frequently less than 1 (see Example 3.2c). Examination of the "MATRICES OF STANDARD NORMAL DEVIATES" shows that there are few extreme values and agreement between model and data seems reasonably good. This is borne out by the goodness of fit test which yields (after much pooling) a chi-square value of 64.79 with 57 df. If H_{02} is true, there is a better than 20% chance of a chi-square value at least this extreme (64.79) so there is no reason to suspect that the model is incorrect. The likelihood ratio test comparing H_{02} with H_1 (inadvertently not photographed as part of Example 3.2c) also suggests that the model under H_{02} is adequate for these data ($\chi^2=15.14$, 11 df, $P<0.18$). This test is discussed in Section 3.7.

3.4 The Model Under H_{01}

The model discussed in this section is simple but very restrictive. As under H_{02}, the young and adult annual survival rates are assumed to be constant, but in addition young and adult recovery rates are also assumed to be constant. In most situations this last assumption is not appropriate and consequently the model is inadequate (see Examples 3.3 and 3.5b).

The assumptions of H_{01} are:

(1) Annual recovery and survival rates are age-dependent for the first year of life only; and
(2) annual recovery and survival rates are otherwise constant from year to year.

The parameters of the model under H_{01} are:

f' = constant annual recovery rate for young,
S' = constant annual survival rate for young,
f = constant annual recovery rate for adults,
S = constant annual survival rate for adults.

The four parameters f, f', S, S' are each estimable.

Tables of expected numbers of band recoveries, in terms of N_i, M_i, f, f', S, S', are used to express the structure of this model.

Table 3.5. *Expected numbers of band recoveries under H_{01} for a banding study with $k=3, \ell=5, s=2$.*

Year banded	Number banded	Year of recovery 1	2	3	4	5
		Birds banded and released as adults				
1	N_1	N_1f	N_1Sf	N_1SSf	N_1SSSf	N_1SSSSf
2	N_2		N_2f	N_2Sf	N_2SSf	N_2SSSf
3	N_3			N_3f	N_3Sf	N_3SSf
		Birds banded and released as young				
1	M_1	M_1f'	$M_1S'f$	$M_1S'Sf$	$M_1S'SSf$	$M_1S'SSSf$
2	M_2		M_2f'	$M_2S'f$	$M_2S'Sf$	$M_2S'SSf$
3	M_3			M_3f'	$M_3S'f$	$M_3S'Sf$

Example 3.2a

MALE BLUE WING TEAL BANDED PRESEASON IN SASKATCHEWAN, 1962–68

ADULTS INPUT MATRIX

1962	1033.	3.	3.	3.	3.	1.	3.	0.	0.	0.	0.	0.	0.
1963	2233.	0.	14.	5.	11.	6.	5.	2.	0.	1.	0.	0.	0.
1964	2658.	0.	0.	17.	18.	7.	10.	1.	7.	1.	1.	0.	0.
1965	1705.	0.	0.	0.	9.	14.	6.	2.	0.	4.	0.	0.	0.
1966	4699.	0.	0.	0.	0.	44.	23.	5.	9.	2.	5.	0.	4.
1967	4542.	0.	0.	0.	0.	0.	47.	14.	19.	19.	8.	5.	2.
1968	2892.	0.	0.	0.	0.	0.	0.	11.	25.	12.	4.	2.	2.

YOUNG INPUT MATRIX

1962	910.	6.	2.	1.	1.	0.	2.	1.	0.	0.	0.	0.	0.
1963	1157.	0.	11.	5.	6.	1.	1.	1.	1.	0.	0.	0.	1.
1964	1394.	0.	0.	19.	4.	4.	4.	0.	0.	1.	1.	0.	0.
1965	3554.	0.	0.	0.	65.	25.	8.	4.	2.	4.	4.	1.	0.
1966	4849.	0.	0.	0.	0.	65.	17.	2.	10.	6.	2.	3.	1.
1967	2555.	0.	0.	0.	0.	0.	52.	9.	8.	3.	4.	2.	1.
1968	305.	0.	0.	0.	0.	0.	0.	3.	1.	0.	1.	0.	0.

BASIC SUBTOTALS

I	RROW(I)	RCOL(I)	CROW(I)	QCOL(I)	T(I)	U(I)	W(I)	Z(I)
1	16.00	3.00	13.00	6.00	16.00	13.00	3.00	0.0
2	44.00	17.00	27.00	13.00	57.00	34.00	19.00	13.00
3	62.00	25.00	33.00	25.00	102.00	54.00	31.00	45.00
4	35.00	41.00	113.00	76.00	112.00	142.00	52.00	92.00
5	92.00	72.00	106.00	95.00	163.00	172.00	102.00	89.00
6	114.00	94.00	79.00	84.00	205.00	156.00	126.00	127.00
7	56.00	35.00	5.00	20.00	167.00	77.00	52.00	156.00
					132.00	57.00		187.00

THE HYPOTHESIS H02

ASSUMPTIONS: (1) YOUNG AND ADULTS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES

(2) SURVIVAL RATES ARE OTHERWISE CONSTANT FROM YEAR TO YEAR

(3) RECOVERY RATES ARE YEAR-SPECIFIC

PARAMETERS:

S = CONSTANT ANNUAL SURVIVAL RATE FOR ADULTS
F(I) = BAND RECOVERY RATE IN YEAR I FOR ADULTS
S' = CONSTANT ANNUAL SURVIVAL RATE FOR YOUNG
F'(I) = BAND RECOVERY RATE IN YEAR I FOR YOUNG

STRUCTURE OF THE MODEL UNDER H02 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS				BANDED AS YOUNG			
N(1)F(1)	N(1)SF(2)	N(1)SSF(3)	N(1)SSSF(4)	M(1)F'(1)	M(1)S'F(2)	M(1)S'SF(3)	M(1)S'SSF(4)
	N(2)F(2)	N(2)SF(3)	N(2)SSF(4)		M(2)F'(2)	M(2)S'F(3)	M(2)S'SF(4)
		N(3)F(3)	N(3)SF(4)			M(3)F'(3)	M(3)S'F(4)

ESTIMATES UNDER H02

F(I) RECOVERY RATE FOR ADULTS / F'(I) RECOVERY RATE FOR YOUNG

I	YEAR	F(I) ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	F'(I) ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1962	0.0029	0.0017	-0.0004 - 0.0062	0.0066	0.0027	0.0013 - 0.0118
2	1963	0.0057	0.0013	0.0031 - 0.0083	0.0095	0.0029	0.0039 - 0.0151
3	1964	0.0058	0.0011	0.0038 - 0.0079	0.0136	0.0031	0.0075 - 0.0197
4	1965	0.0090	0.0013	0.0065 - 0.0115	0.0183	0.0022	0.0139 - 0.0227
5	1966	0.0101	0.0011	0.0080 - 0.0122	0.0134	0.0016	0.0101 - 0.0166
6	1967	0.0095	0.0009	0.0076 - 0.0113	0.0204	0.0028	0.0149 - 0.0259
7	1968	0.0041	0.0006	0.0029 - 0.0053	0.0098	0.0056	-0.0013 - 0.0208

	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$	= 0.0067	0.0005	0.0058 - 0.0077	$\bar{F}'$	= 0.0131	0.0012	0.0107 - 0.0155

S SURVIVAL RATE FOR ADULTS / S' SURVIVAL RATE FOR YOUNG

S ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	S' ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.6356	0.0240	0.5885 - 0.6827	0.4859	0.0460	0.3957 - 0.5760

Example 3.2b

SELECTED ESTIMATED COVARIANCES AND CORRELATIONS UNDER H02

I	YEAR	COV(F(I),S)	CORR(F(I),S)	COV(F(I),S')	CORR(F(I),S')
1	1962	-0.00000007	-0.0017	0.00000001	0.0002
2	1963	-0.00000133	-0.0424	-0.00000350	-0.0581
3	1964	-0.00000284	-0.1125	-0.00000451	-0.0933
4	1965	-0.00000768	-0.2464	-0.00000973	-0.1630
5	1966	-0.00000690	-0.2705	-0.00001213	-0.2482
6	1967	-0.00000766	-0.3405	-0.00001364	-0.3167
7	1968	-0.00000484	-0.3268	-0.00000630	-0.2220

I	YEAR	COV(F'(I),S)	CORR(F'(I),S)	COV(F'(I),S')	CORR(F'(I),S')
1	1962	-0.00000008	-0.0013	-0.00000023	-0.0019
2	1963	-0.00000009	-0.0014	-0.00000035	-0.0027
3	1964	-0.00000008	-0.0010	-0.00000054	-0.0038
4	1965	-0.00000004	-0.0008	-0.00000067	-0.0065
5	1966	0.00000003	0.0008	-0.00000042	-0.0055
6	1967	0.00000017	0.0025	-0.00000053	-0.0041
7	1968	0.00000020	0.0015	-0.00000029	-0.0011

I	YEAR	COV(F(I),F'(I))	CORR(F(I),F'(I))	COV(F(I+1),F'(I))	CORR(F(I+1),F'(I))
1	1962	0.00000000	0.0000	-0.00000000	-0.0014
2	1963	0.00000000	0.0002	-0.00000000	-0.0013
3	1964	0.00000000	0.0004	-0.00000001	-0.0018
4	1965	0.00000000	0.0011	-0.00000000	-0.0021
5	1966	0.00000000	0.0010	-0.00000000	-0.0017
6	1967	0.00000000	0.0004	-0.00000000	-0.0019
7	1968	-0.00000000	-0.0002	-0.00000001	-0.0013

I	YEAR	COV(F(I),F'(I+1))	CORR(F(I),F'(I+1))	COV(F'(I),F'(I+1))	CORR(F'(I),F'(I+1))
1	1962	0.00000000	0.0000	0.00000000	0.0000
2	1963	0.00000000	0.0002	0.00000000	0.0000
3	1964	0.00000000	0.0006	0.00000000	0.0000
4	1965	0.00000000	0.0006	0.00000000	0.0000
5	1966	0.00000000	0.0003	0.00000000	0.0000
6	1967	-0.00000000	-0.0001	0.00000000	0.0000

I	YEAR	COV(F(I),F(I+1))	CORR(F(I),F(I+1))	COV(F(I),F(I+2))	CORR(F(I),F(I+2))
1	1962	-0.00000000	-0.0014	-0.00000000	-0.0006
2	1963	0.00000001	0.0066	0.00000003	0.0159
3	1964	0.00000005	0.0354	0.00000005	0.0465
4	1965	0.00000013	0.0937	0.00000015	0.1205
5	1966	0.00000015	0.1501	0.00000008	0.1276
6	1967	0.00000009	0.1611		

COV(S,S')	CORR(S,S')
0.00012635	0.1144

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

NUMBER OF ITERATIONS COMPLETED = 4

Example 3.2c

MALE BLUE WING TEAL BANDED PRESEASCN IN SASKATCHEWAN, 1962-68

H02

MATRIX OF DATA VALUES -- ADULTS

1	3.00	3.00	3.00	3.00	1.00	3.00	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	14.00	5.00	11.00	6.00	5.00	2.00	0.0	1.00	0.0	0.0	0.0
3	0.0	0.0	17.00	18.00	7.0C	10.00	1.00	7.0C	1.00	1.00	0.0	0.0
4	0.0	0.0	0.0	9.00	14.0C	6.00	2.00	0.C	4.0C	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	44.0C	23.00	5.00	9.00	2.00	5.00	0.0	4.00
6	0.0	0.0	0.0	0.0	C.0	47.00	14.00	19.0C	19.00	8.00	5.00	2.00
7	0.0	C.0	0.0	0.0	C.0	0.0	11.00	25.00	12.00	4.00	2.00	2.00

MATRIX OF EXPECTED VALUES -- ADULTS

1	3.00	3.75	2.43	2.39	1.7C	1.01	0.28	0.44	0.28	0.16	0.07	0.06
2	0.0	12.74	8.26	8.12	5.78	3.45	0.96	1.48	0.96	0.54	0.23	0.20
3	0.0	0.0	15.46	15.21	1C.83	6.46	1.79	2.77	1.79	1.01	0.44	0.37
4	0.0	0.0	0.0	15.35	1C.93	6.52	1.81	2.8C	1.81	1.02	0.44	0.38
5	0.0	0.0	0.0	0.0	47.39	28.26	7.84	12.14	7.84	4.44	1.92	1.63
6	0.0	0.0	0.0	0.0	0.0	42.98	11.92	18.46	11.93	6.75	2.93	2.48
7	0.0	0.0	0.0	0.0	C.0	0.0	11.94	18.49	11.95	6.76	2.93	2.48

MATRIX OF STANDARD NORMAL DEVIATES -- ADULTS

1	-0.00	-0.39	0.37	0.40	-C.54	1.97	-0.53	-0.66	-0.53	-0.40	-0.26	-0.24
2	0.0	0.35	-1.14	1.01	0.09	0.84	1.07	-1.22	0.04	-0.74	-0.48	-0.45
3	0.0	C.0	0.39	0.72	-1.17	1.40	-0.59	2.54	-0.59	-0.01	-0.66	-0.61
4	0.0	0.0	0.0	-1.63	C.93	-0.20	0.14	-1.67	1.63	-1.01	-0.67	-0.61
5	0.0	0.0	0.0	0.0	-C.49	-0.99	-1.01	-0.5C	-2.09	0.27	-1.39	1.86
6	0.0	C.0	0.0	0.0	0.0	0.62	0.60	0.13	2.05	0.48	1.21	-0.30
7	0.0	C.0	0.0	0.0	C.0	0.0	-0.27	1.52	0.01	-1.06	-0.54	-0.31

MATRIX OF DATA VALUES -- YOUNG

1	6.0C	2.0C	1.00	1.00	0.0	2.00	1.00	0.0	0.0	0.0	0.0	0.0
2	0.0	11.00	5.00	6.00	1.00	1.00	1.00	1.0C	0.0	0.0	0.0	1.00
3	0.0	0.0	19.00	4.00	4.00	4.0C	0.0	0.0	1.00	1.00	0.0	0.0
4	0.0	0.0	0.0	65.00	25.00	8.00	4.00	2.00	4.0C	4.00	1.00	0.0
5	0.0	0.0	0.0	0.0	65.00	17.00	2.00	10.00	6.0C	2.00	3.00	1.00
6	0.0	0.0	0.0	0.0	0.0	52.00	9.00	8.00	3.00	4.00	2.00	1.00
7	0.0	0.0	0.0	0.0	0.0	0.0	3.00	1.0C	0.0	1.00	0.0	0.0

MATRIX OF EXPECTED VALUES -- YOUNG

1	5.99	2.52	1.63	1.61	1.14	0.68	0.19	0.29	0.19	0.11	0.05	0.04
2	0.0	11.04	3.27	3.22	2.29	1.37	0.38	0.59	0.38	0.21	0.09	0.08
3	0.0	0.0	18.97	6.10	4.34	2.59	0.72	1.11	0.72	0.41	0.18	0.15
4	0.0	0.0	0.0	65.13	17.42	10.39	2.88	4.46	2.88	1.63	0.71	0.60
5	0.0	0.0	0.0	0.0	64.87	22.30	6.18	9.57	6.19	3.50	1.52	1.28
6	0.0	0.0	0.0	0.0	0.0	52.07	5.13	7.94	5.13	2.90	1.26	1.06
7	0.0	0.0	0.0	0.0	C.0	0.0	2.99	1.49	0.96	0.55	0.24	0.20

MATRIX OF STANDARD NORMAL DEVIATES -- YOUNG

1	0.00	-0.33	-0.50	-0.48	-1.07	1.59	1.86	-0.54	-0.44	-0.33	-0.22	-0.20
2	0.0	-0.01	0.96	1.55	-C.85	-0.31	1.01	0.54	-0.62	-0.46	-0.30	3.28
3	0.0	0.0	0.01	-0.85	-C.16	0.88	-0.85	-1.05	0.33	0.93	-0.42	-0.39
4	0.0	0.0	0.0	-0.02	1.82	-0.74	0.66	-1.17	0.66	1.85	0.35	-0.77
5	0.0	0.0	0.0	0.0	0.02	-1.12	-1.68	0.14	-0.08	-0.80	1.20	-0.25
6	0.0	0.0	0.0	0.0	0.0	-0.01	1.71	0.02	-0.94	0.64	0.66	-0.06
7	0.0	C.0	0.0	0.0	0.0	0.0	0.01	-0.4C	-0.98	0.62	-0.49	-0.45

THESE STANDARD NORMAL DEVIATES ARE USEFUL IN EXAMINING THE AGREEMENT BETWEEN THE MODEL AND THE OBSERVED DATA IN A PARTICULAR CELL. FOR EXAMPLE, A VALUE OF SAY -5 FOR A PARTICULAR CELL MAY INDICATE AN UNUSUAL OBSERVATION OR, PERHAPS, A MISTAKE WAS MADE IN SUMMARIZING THE DATA. IF THE MODEL IS CORRECT, ABOUT 95% OF THESE STATISTICS SHOULD LIE WITHIN THE INTERVAL -2 TO 2.

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H02.

CHI-SQUARE VALUE = 64.79
DEGREES OF FREEDOM = 57
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 64.79 = 0.22335

Estimation of Parameters

Simple expressions cannot be obtained for the ML estimators of the four parameters f, f', S, and S' and evaluation of the estimates for a given data set using a desk calculator would be extremely tedious. However, estimates are easily obtained by using program BROWNIE as illustrated in Example 3.3 (see Appendices A and B for a discussion of the method used in the program). Estimates of the standard errors and confidence intervals are also printed, as well as estimates of the covariances and correlations between the estimators.

No goodness of fit test is computed for this model in program BROWNIE because the performance of this model is on the whole very poor, and the H_{02} model will almost always be preferred. That is, comparison of H_{01} with H_{02} (by means of a likelihood ratio test) almost always results in a clear-cut rejection of H_{01}, in favor of H_{02}, and testing fit to H_{01} then seems unnecessary.

An Example

The blue-winged teal data of Example 3.2 are also used here to illustrate the analysis under H_{01}. The sample output is self-explanatory. Note again the low recovery rates for these data, and the age-dependence of both recovery and survival rates.

The likelihood ratio test (discussed further in Section 3.7) comparing H_{01} with H_{02} is very significant, and therefore the model under H_{01} is rejected in favor of the model under H_{02}; thus the assumption that adult and young recovery rates are constant from year to year is also rejected.

3.5 The Model Under H_2

The reasons that led to considering Model 0 as an extension of Model 1 in Chapter 2 also suggest a similar extension of the H_1 model of this chapter. As discussed in Section 2.5, reporting rates may be different for hunters near banding sites, being increased by band solicitation or decreased by hunters who are more accustomed to seeing bands. A difference in the reporting rate near banding sites affects primarily newly banded birds that are more concentrated near banding sites at the beginning of the hunting season than birds banded in previous years.

In defining the model under H_2, we note that there are many similarities between the H_2 model of this chapter and Model 0 of Chapter 2. Thus the assumptions of H_2 are appropriate in the bird banding context, but may not be valid for similar studies on other types of populations, such as fish-tagging experiments. Also, there are other sets of assumptions which give rise to models that cannot be distinguished from the model under H_2 on the basis of the type of data collected. These points are discussed further below.

The assumptions of H_2 are:

(1) Annual survival, harvest and reporting rates are year-specific;

(2) annual survival and harvest rates are age-dependent for the first year of life only; and

(3) in any year, the reporting rate for newly released birds is different from that for survivors of previous releases.

Assumptions 2 and 3 lead to defining three recovery rates for the model under H_2. The parameters of this model are:

f_i' = recovery rate in year i for birds banded and released as young in year i, $i=1,\ldots,k$,

S_i' = survival rate for young in year i, $i=\begin{cases}1,\ldots,k-1 \text{ if } \ell=k\\ 1,\ldots,k \text{ if } \ell>k,\end{cases}$

f_i''' = recovery rate in year i for adults released in year i, $i=1,\ldots,k$,

f_i = recovery rate in year i for survivors of birds released before year i, $i=2,\ldots,\ell$,

S_i = survival rate for adults in year i, $i=1,\ldots,\ell-1$.

Note that although assumption 3 above applies to the reporting rate for bands taken from young as well as from adults, the rate parameters defined for young under H_2 are the same as those under H_1. This is because the actual reporting and harvest rates are not separately identifiable, so that the harvest rate for young and the different reporting rate for new releases are both reflected in the recovery rate f_i'.

Tables of expected numbers of band recoveries are used to express the structure of this model in terms of N_i, M_i, f_i''', f_i, f_i', S_i, and S_i'. Note that, as suggested by the paragraph above, the expected recoveries for birds released as young are the same under H_2 and H_1 (cf. Tables 3.6 and 3.3).

Example 3.3

MALE BLUE WING TEAL BANDEC PRESEASCN IN SASKATCHEWAN, 1962-68

THE HYPOTHESIS H01

ASSUMPTIONS: (1) YOUNG AND ADULTS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES
(2) OTHERWISE, SURVIVAL AND RECOVERY RATES ARE CONSTANT FRCM YEAR TO YEAR

PARAMETERS:

S = CONSTANT ANNUAL SURVIVAL RATE FOR ADULTS
F = CONSTANT BAND RECOVERY RATE FOR ADULTS
S' = CONSTANT ANNUAL SURVIVAL RATE FOR YOUNG
F' = CONSTANT BAND RECOVERY RATE FOR YOUNG

STRUCTURE OF THE MODEL UNDER H01 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS				BANDED AS YOUNG			
N(1)F	N(1)SF	N(1)SSF	N(1)SSSF	M(1)F'	M(1)S'F	M(1)S'SF	M(1)S'SSF
	N(2)F	N(2)SF	N(2)SSF		M(2)F'	M(2)S'F	M(2)S'SF
		N(3)F	N(3)SF			M(3)F'	M(3)S'F

ESTIMATES UNDER H01

F RECOVERY RATE FOR ADULTS

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.0077	0.0005	0.0068 - 0.0086

F' RECOVERY RATE FOR YOUNG

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.0150	0.0010	0.0130 - 0.0170

S SURVIVAL RATE FOR ADULTS

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.6492	0.0152	0.6193 - 0.6791

S' SURVIVAL RATE FOR YOUNG

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.5007	0.0468	0.4090 - 0.5923

ESTIMATED COVARIANCES AND CORRELATIONS UNDER H01

COV(F,F') -0.00000000	CORR(F,F') -0.0000	COV(F,S') -0.00000917	CORR(F,S') -0.4172
COV(F,S) -0.00000438	CORR(F,S) -0.6112	COV(S,F') -0.00000000	CORR(S,F') -0.0000
COV(S,S') 0.00000875	CORR(S,S') 0.0123	COV(F',S') -0.00000051	CORR(F',S') -0.0109

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

NUMBER OF ITERATIONS COMPLETED = 4

LIKELIHOOD RATIO TEST OF H01 VS H02.

THIS TEST COMPARES THE MODEL UNDER H01 WITH THAT UNDER H02 AND THUS TESTS THE ASSUMPTION THAT ADULT AND YOUNG RECOVERY RATES ARE CONSTANT FROM YEAR TO YEAR. A 'LARGE' CHI-SQUARE VALUE INDICATES THAT H02 BETTER DESCRIBES THE DATA AND THAT RECOVERY RATES ARE NOT CONSTANT FROM YEAR TO YEAR.

CHI-SQUARE VALUE = 72.72
DEGREES OF FREEDOM = 17
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 72.72 = 0.00000

Table 3.6. *Expected numbers of band recoveries under H_2 for a banding study with $k=3, \ell=5, s=2$.*

Year banded	Number banded	Year of recovery 1	2	3	4	5
				Birds banded and released as adults		
1	N_1	N_1f_1'''	$N_1S_1f_2$	$N_1S_1S_2f_3$	$N_1S_1S_2S_3f_4$	$N_1S_1S_2S_3S_4f_5$
2	N_2		N_2f_2'''	$N_2S_2f_3$	$N_2S_2S_3f_4$	$N_2S_2S_3S_4f_5$
3	N_3			N_3f_3'''	$N_3S_3f_4$	$N_3S_3S_4f_5$
				Birds banded and released as young		
1	M_1	M_1f_1'	$M_1S_1'f_2$	$M_1S_1'S_2f_3$	$M_1S_1'S_2S_3f_4$	$M_1S_1'S_2S_3S_4f_5$
2	M_2		M_2f_2'	$M_2S_2'f_3$	$M_2S_2'S_3f_4$	$M_2S_2'S_3S_4f_5$
3	M_3			M_3f_3'	$M_3S_3'f_4$	$M_3S_3'S_4f_5$

Estimation of Parameters

ML estimators of the different recovery rates are:

$$\hat{f}_i=\frac{R_{i\cdot}-R_{ii}}{N_i}\frac{W_i-R_{ii}}{T_i+U_i-Q_{i\cdot}-R_{i\cdot}-W_i+R_{ii}} \quad ,i=\begin{cases}2,\ldots,k-1 \text{ if } \ell=k\\ 2,\ldots,k \text{ if } \ell>k\end{cases}$$

$$\hat{f}_i'''=\frac{R_{ii}}{N_i} \quad ,i=1,\ldots,k,$$

$$\hat{f}_i'=\frac{Q_{ii}}{M_i} \quad ,i=1,\ldots,k.$$

The data of Table 3.2 give

$$\hat{f}_1'''=\frac{R_{11}}{N_1}=\frac{10}{231}=0.0433,$$

$$\hat{f}_1'=\frac{Q_{11}}{M_1}=\frac{83}{962}=0.0863$$

$$\hat{f}_2=\frac{R_{2\cdot}-R_{22}}{N_2}\frac{W_2-R_{22}}{T_2+U_2-Q_{2\cdot}-R_{2\cdot}-W_2+R_{22}}=\frac{(131-58)\times(106-58)}{649\times(250-131-106+58)}=0.0760$$

$$\hat{f}_2'''=\frac{58}{649}=0.0894,$$

$$\hat{f}_2'=\frac{103}{702}=0.1467$$

$$\hat{f}_3=\frac{(161-54)\times(120-54)}{885\times(370-161-120+54)}=0.0558.$$

Bias-adjusted ML estimators of survival rates are:

$$\tilde{S}_i=\frac{R_{i\cdot}-R_{ii}}{N_i}\frac{N_{i+1}+1}{R_{i+1\cdot}-R_{i+1,i+1}+1}\left(\frac{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}-W_{i+1}+R_{i+1,i+1}}{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}}\right) \quad ,i=\begin{cases}1,\ldots,k-2 \text{ if } \ell=k\\ 1,\ldots,k-1 \text{ if } \ell>k\end{cases}$$

$$\tilde{S}_i'=\frac{Q_{i\cdot}-Q_{ii}}{M_i}\frac{N_{i+1}+1}{R_{i+1\cdot}-R_{i+1,i+1}+1}\left(\frac{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}-W_{i+1}+R_{i+1,i+1}}{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}}\right) \quad ,i=\begin{cases}1,\ldots,k-2 \text{ if } \ell=k\\ 1,\ldots,k-1 \text{ if } \ell>k.\end{cases}$$

The data of Table 3.2 give

$$\tilde{S}_1 = \frac{R_{1\cdot} - R_{11}}{N_1} \frac{N_2 + 1}{R_{2\cdot} - R_{22} + 1} \left(\frac{T_2 + U_2 - Q_{2\cdot} - R_{2\cdot} - W_2 + R_{22}}{T_2 + U_2 - Q_{2\cdot} - R_{2\cdot}} \right) = \frac{(37 - 10) \times (649 + 1) \times (250 - 131 - 106 + 58)}{231 \times (131 - 58 + 1) \times (250 - 131)} = 0.6126$$

$$\tilde{S}_1' = \frac{Q_{1\cdot} - Q_{11}}{M_1} \frac{N_2 + 1}{R_{2\cdot} - R_{22} + 1} \left(\frac{T_2 + U_2 - Q_{2\cdot} - R_{2\cdot} - W_2 + R_{22}}{T_2 + U_2 - Q_{2\cdot} - R_{2\cdot}} \right) = \frac{(175 - 83) \times (649 + 1) \times (250 - 131 - 106 + 58)}{962 \times (131 - 58 + 1) \times (250 - 131)} = 0.5012.$$

These calculations can be compared with the complete evaluation of estimates in the printout from the FORTRAN program presented in Example 3.5i and 3.5j.

For completeness we define the unadjusted ML estimators of survival and additional ML estimators, most of which are appropriate when the data are nontriangular (i.e., when $\ell > k$). Thus

$$\hat{S}_i = \frac{R_{i\cdot} - R_{ii}}{N_i} \frac{N_{i+1}}{R_{i+1\cdot} - R_{i+1,i+1}} \left(\frac{T_{i+1} + U_{i+1} - Q_{i+1\cdot} - R_{i+1\cdot} - W_{i+1} + R_{i+1,i+1}}{T_{i+1} + U_{i+1} - Q_{i+1\cdot} - R_{i+1\cdot}} \right), \quad i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k. \end{cases}$$

$$\hat{S}_i' = \frac{Q_{i\cdot} - Q_{ii}}{M_i} \frac{N_{i+1}}{R_{i+1\cdot} - R_{i+1,i+1}} \left(\frac{T_{i+1} + U_{i+1} - Q_{i+1\cdot} - R_{i+1\cdot} - W_{i+1} + R_{i+1,i+1}}{T_{i+1} + U_{i+1} - Q_{i+1\cdot} - R_{i+1\cdot}} \right), \quad i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k. \end{cases}$$

In addition, if $\ell = k$,

$$\widehat{S_{k-1} f_k} = \frac{R_{k-1\cdot} - R_{k-1,k-1}}{N_{k-1}}$$

$$\widehat{S_{k-1}' f_k} = \frac{Q_{k-1\cdot} - Q_{k-1,k-1}}{M_{k-1}}$$

and if $\ell = k + 1$,

$$\widehat{S_k f_{k+1}} = \frac{R_{k\cdot} - R_{kk}}{N_k}$$

$$\widehat{S_k' f_{k+1}} = \frac{Q_{k\cdot} - Q_{kk}}{M_k}$$

and finally if $\ell > k + 1$,

$$\widehat{S_k \cdots S_{k+i-1} f_{k+i}} = \frac{R_{\cdot k+i} + Q_{\cdot k+i}}{T_{k+1} + U_{k+1}} \frac{R_{k\cdot} - R_{kk}}{N_k}, \quad i = 1, \ldots, s,$$

$$\widehat{S_k' f_{k+1}} = \frac{R_{\cdot k+1} + Q_{\cdot k+1}}{T_{k+1} + U_{k+1}} \frac{Q_{k\cdot} - Q_{kk}}{M_k}$$

$$\widehat{S_k' S_{k+1} \cdots S_{k+i-1} f_{k+1}} = \frac{R_{\cdot k+i} + Q_{\cdot k+i}}{T_{k+1} + U_{k+1}} \frac{Q_{k\cdot} - Q_{kk}}{M_k}, \quad i = 2, \ldots, s.$$

These additional estimates are used in program BROWNIE in obtaining the matrices of "expected values" and of "standard normal deviates," but they are not all contained in the printout because they are not of biological interest. Examining the output in Example 3.5j shows that, for the data of Table 3.2 where $k = \ell = 9$, and $s = 0$,

$$\widehat{S_{k-1} f_k} = \widehat{S_8 f_9} = \frac{R_{8\cdot} - R_{88}}{N_8} = \frac{R_{89}}{N_8} = \frac{22}{938} = 0.0235.$$

However, $\widehat{S_{k-1}' f_k} = \frac{25}{906} = 0.0276$ is not printed.

Sampling Variances, Standard Errors, and Confidence Intervals

Estimators of the large-sample variances of $\hat{f}_i$, $\hat{f}_i'''$, $\hat{f}_i'$, $\tilde{S}_i$ and $\tilde{S}_i'$ are given below. The notation is similar to that introduced in Section 3.2 for the H_1 model. These variance estimators are used as described in Section 3.2 to obtain estimates of the corresponding standand errors and 95% confidence intervals, all of which are contained in the printout of the FORTRAN program BROWNIE (see Example 3.5).

$$\mathrm{var}(\hat{f}_i) = (\hat{f}_i)^2\left[\frac{1}{R_{i\cdot}-R_{ii}} - \frac{1}{N_i} + \frac{1}{T_i+U_i-Q_{i\cdot}-R_{i\cdot}-W_i+R_{ii}} + \frac{1}{W_i-R_{ii}}\right], i=\begin{cases}2,\ldots,k-1 & \text{if } \ell=k\\ 2,\ldots,k & \text{if } \ell>k,\end{cases}$$

$$\mathrm{var}(\hat{f}_i''') = \hat{f}_i'''(1-\hat{f}_i''')/N_i, \quad i=1,\ldots,k,$$

$$\mathrm{var}(\hat{f}_i') = \hat{f}_i'(1-\hat{f}_i')/M_i, \quad i=1,\ldots,k,$$

$$\mathrm{var}(\tilde{S}_i) = (\tilde{S}_i)^2\left[\frac{1}{R_{i\cdot}-R_{ii}} - \frac{1}{N_i} + \frac{1}{R_{i+1\cdot}-R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right.$$
$$\left. + \frac{1}{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}-W_{i+1}+R_{i+1,i+1}} - \frac{1}{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}}\right], i=\begin{cases}1,\ldots,k-2 & \text{if } \ell=k\\ 1,\ldots,k-1 & \text{if } \ell>k,\end{cases}$$

$$\mathrm{var}(\tilde{S}_i') = (\tilde{S}_i')^2\left[\frac{1}{Q_{i\cdot}-Q_{ii}} - \frac{1}{M_i} + \frac{1}{R_{i+1\cdot}-R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right.$$
$$\left. + \frac{1}{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}-W_{i+1}+R_{i+1,i+1}} - \frac{1}{T_{i+1}+U_{i+1}-Q_{i+1\cdot}-R_{i+1\cdot}}\right], i=\begin{cases}1,\ldots,k-2 & \text{if } \ell=k\\ 1,\ldots,k-1 & \text{if } \ell>k.\end{cases}$$

The computation of these variances and corresponding confidence intervals without using the FORTRAN program is somewhat tedious, as we illustrate below, again using the data of Table 3.2

$$\mathrm{var}(\hat{f}_3) = (\hat{f}_3)^2\left[\frac{1}{R_{3\cdot}-R_{33}} - \frac{1}{N_3} + \frac{1}{T_3+U_3-Q_{3\cdot}-R_{3\cdot}-W_3+R_{33}} + \frac{1}{W_3-R_{33}}\right]$$

$$= (0.0558)^2\left[\frac{1}{161-54} - \frac{1}{885} + \frac{1}{370-161-120+54} + \frac{1}{120-54}\right] = 0.00009453,$$

$$\mathrm{se}(\hat{f}_3) = 0.0097,$$

$$1.96\times \mathrm{se}(\hat{f}_3) = 0.0190$$

and the 95 % confidence interval for f_3 is (0.0368, 0.0748). Comparison with the corresponding results in the printout of Example 3.5i shows a slight difference in the confidence interval obtained there, which is again due to the greater accuracy of the calculations performed by the computer.

Sampling Covariances and Correlations

Formulae for obtaining estimates of the covariances between the estimators $\hat{f}_i, \hat{f}_i'''$, $\hat{f}_i'$, $\tilde{S}_i$, and $\tilde{S}_i'$ are given below, again using the notation of Section 3.2. Estimates of the correlations between corresponding pairs of estimators are obtained using the covariances and variances above in the manner described in Section 3.2 for the H_1 estimators. Use of the FORTRAN program is recommended to avoid the time-consuming computations involved in the evaluation of all the formulae below which are presented here mainly for reference purposes or for use by the reader with a single data set with small k, (e.g., $k=4$ or 5).
Thus,

$$\text{cov}(\hat{f}_i''',\hat{f}_i) = -\hat{f}_i'''\hat{f}_i/N_i \qquad , i = \begin{cases} 2,\ldots,k-1 \text{ if } \ell = k \\ 2,\ldots,k \quad \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\hat{f}_i''',\tilde{S}_i) = -\hat{f}_i'''\tilde{S}_i/N_i \qquad , i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\hat{f}_i'''{}_1,\tilde{S}_i) = \hat{f}_i'''{}_1\tilde{S}_i/N_{i+1} \qquad , i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\hat{f}_{i+1}''',\tilde{S}_i') = \hat{f}_i'''{}_1\tilde{S}_i'/N_{i+1} \qquad , i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\hat{f}_i,\tilde{S}_i) = \hat{f}_i\tilde{S}_i\left[\frac{1}{R_{i\cdot} - R_{ii}} - \frac{1}{N_i}\right] \qquad , i = \begin{cases} 2,\ldots,k-2 \text{ if } \ell = k \\ 2,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\hat{f}_{i+1},\tilde{S}_i) = -\hat{f}_{i+1}\tilde{S}_i\left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right] \qquad , i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\hat{f}_{i+1},\tilde{S}_i') = -\hat{f}_{i+1}\tilde{S}_i'\left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right] \qquad , i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\tilde{S}_i,\tilde{S}_{i+1}) = -\tilde{S}_i\tilde{S}_{i+1}\left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right] \qquad , i = \begin{cases} 1,\ldots,k-3 \text{ if } \ell = k \\ 1,\ldots,k-2 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\tilde{S}_i,\tilde{S}_i') = \tilde{S}_i\tilde{S}_i'\left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}} + \frac{1}{T_{i+1} + U_{i+1} - Q_{i+1\cdot} - R_{i+1\cdot} - W_{i+1} + R_{i+1,i+1}} - \frac{1}{T_{i+1} + U_{i+1} - Q_{i+1\cdot} - R_{i+1\cdot}}\right] \qquad , i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\tilde{S}_{i+1},\tilde{S}_i') = -\tilde{S}_{i+1}\tilde{S}_i'\left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right] \qquad , i = \begin{cases} 1,\ldots,k-3 \text{ if } \ell = k \\ 1,\ldots,k-2 \text{ if } \ell > k, \end{cases}$$

$$\text{cov}(\hat{f}_i',\tilde{S}_i) = -\hat{f}_i'\tilde{S}_i'/M_i \qquad , i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

For the mallard data of Table 3.2 we have obtained above $\hat{f}_3 = 0.0558$, se$(\hat{f}_3) = 0.0097$, and from Example 3.5i we obtain $\tilde{S}_3 = 0.7427$, and se$(\tilde{S}_3) = 0.1137$. Thus, as shown in example 3.5k.

$$\text{cov}(\hat{f}_3,\tilde{S}_3) = \hat{f}_3\tilde{S}_3\left[\frac{1}{R_{3\cdot} - R_{33}} - \frac{1}{N_3}\right] = (0.0558) \times (0.7427) \times \left[\frac{1}{161 - 54} - \frac{1}{885}\right] = 0.00034049.$$

The estimate of the correlation between $\hat{f}_3$ and $\tilde{S}_3$ using the method described in Section 3.2, is

$$\text{corr}(\hat{f}_3,\tilde{S}_3) = \frac{\text{cov}(\hat{f}_3,\tilde{S}_3)}{\text{se}(\hat{f}_3)\text{se}(\tilde{S}_3)} = \frac{0.00034049}{(0.0097) \times (0.1137)} = 0.3087.$$

Again this is slightly different from the corresponding result in the output in Example 3.5k, because of the greater accuracy of the latter.

As stated in Section 3.2, it must be recognized that these correlations reflect a property of the estimators themselves, and not of the unknown parameters. The reader is referred to the discussion in Section 8.4.

Goodness of Fit Test

A goodness of fit test to the model under H_2 can be obtained in the conventional way (described, for example, in Section 3.3) or by the alternative method referred to in Section 3.2 in relation to testing fit to the model under H_1. The method used in program BROWNIE is the latter, and the resulting chi-square value, degrees of freedom, and associated significance level are printed as shown in Example 3.4e. As usual, large chi-square values (values associated with a small probability under H_2) suggest that the model is not appropriate.

In Example 3.5p, the goodness of fit test to the model under H_2 yields a chi-square value of 37.66 with 41 df. The probability under H_2 of observing a value larger than this is 0.62; hence, there is no reason to suspect that the model is inadequate.

Again a rough idea of fit to the model can be obtained by examining the differences (suitably normalized) between each individual observation and the ML estimate of its expectation under H_2. This is done, as described in Sections 3.2 and 3.3, by obtaining matrices of $\mathbf{E}_{ij}$'s, $\mathbf{E}'_{ij}$'s, Z_{ij}'s, and Z'_{ij}'s, using the ML estimates under H_2 of the expected values presented in Table 3.6 (see Example 3.4d).

Proper and Improper Use of the Model under H_2

We now examine models which are based on different sets of assumptions and parameterizations from those of the H_2 model but which, on the basis of the type of data considered in this chapter, are indistinguishable from the H_2 model. Consider a fish-tagging experiment involving young and adults, where tagging is known to affect the recovery and survival of both age groups in the year immediately following tagging. This would give rise to a model where the expected returns for fish tagged as young would be represented as in Table 3.6, but the expected returns for fish tagged as adults would be

$$
\begin{array}{ccccc}
N_1 f_1''' & N_1 S_1''' f_2 & N_1 S_1''' S_2 f_3 & N_1 S_1''' S_2 S_3 f_4 & N_1 S_1''' S_2 S_3 S_4 f_5 \\
 & N_2 f_2''' & N_2 S_2''' f_3 & N_2 S_2''' S_3 f_4 & N_2 S_2''' S_3 S_4 f_5 \\
 & & N_3 f_3''' & N_3 S_3''' f_4 & N_3 S_3''' S_4 f_5 .
\end{array}
$$

Under this model, the parameters f_i, S_i, S_i', and S_i''' are not separately estimable for reasons similar to those given in Section 3.9 in relation to the nonidentifiability of f_i and S_i' if only young are banded. Also the ML estimators of the expected values of R_{ij} and Q_{ij} are exactly the same functions of the data under this model as under the H_2 model. Thus it is not possible to devise a test to distinguish between these models, and the goodness of fit test to H_2 above tests fit to the H_2 model or this alternative model which we will call H_{2a}. Thus if data are analyzed using the FORTRAN program, the tests computed may indicate that the model under H_2 is appropriate when the true model is in fact the H_{2a} model. The H_2 estimates of f_i, S_i, and S_i' will not be valid in this case. Suppose the unknown parameters S_i''' are substantially smaller than the S_i (for instance due to the detrimental effect of tagging) and H_2 estimates of S_i are computed, then these estimates will, on the average, be too small (i.e., negatively biased).

Another model which is also indistinguishable from the H_2 model is characterized by the same expected recoveries for individuals banded as young, but expected recoveries for individuals banded as adults would be

$$
\begin{array}{ccccc}
N_1 f_1 & N_1 S_1''' f_2 & N_1 S_1''' S_2 f_3 & N_1 S_1''' S_2 S_3 f_4 & N_1 S_1''' S_2 S_3 S_4 f_5 \\
 & N_2 f_2 & N_2 S_2''' f_3 & N_2 S_2''' S_3 f_4 & N_2 S_2''' S_3 S_4 f_5 \\
 & & N_3 f_3 & N_3 S_3''' f_4 & N_3 S_3''' S_4 f_5 .
\end{array}
$$

Under this model the parameters are separately estimable, but there does not seem to be a meaningful biological interpretation for this parameterization, and so this model is not given further consideration.

In the bird banding context, banding during the hunting season may give rise to the H_{2a} model. This is because recovery rates for new releases reflect recoveries from part of the hunting season, and survival rates for new releases relate to a period which is less than a year. Thus we would expect f_i''' to be less than f_i, and S_i''' to be higher than S_i in this situation, and the H_{2a} model rather than the H_2 model is appropriate. If the relative difference between S_i''' and S_i is negligible compared with the relative difference between f_i''' and f_i, then the H_2 model may be a reasonable approximation for in-season banding and the H_2 estimates will be only slightly biased. This is not unreasonable, because the relative difference in the period of survival for new releases may be close to 1 month out of 12, whereas the difference in the effective hunting season will be more like 1 month out of 3.

To use the H_2 model as an approximation in this situation, all recoveries must be recorded for the hunting season during which they occur, regardless of whether they occur before or after the time of banding. With the exception of survival rates for newly banded birds, an annual survival rate then applies to the period between the start of one hunting season and the next. If the H_2 model is not a good approximation, i.e., if S_i''' is appreciably greater than S_i, the H_2 estimators $\tilde{S}_i$ will be positively biased (i.e., too large on the average).

This use of the model under H_2 is illustrated with a data set for Canada Geese in Example 3.4 below. Other examples containing discussion of the model under H_2 are Examples 3.5 and 3.6. The former contains the mallard data of Table 3.2 and has already been referred to several times in this section. The latter contains a data set for which the model under H_2 seems to be the most appropriate model.

An Example

The model under H_2 is used as an approximation for analyzing data for Canada geese banded "in-season" (i.e., during the hunting season) at Swan Lake Refuge, Missouri, 1949-57 (Vaught and Kirsch 1966). Appropriate portions of the output from program BROWNIE are presented in Example 3.4.

Banding usually occurred during the final weeks of the hunting season, so recovery rates in any season should be lower for the newly banded adults. In Example 3.4b we see that except for years 1954 and 1956, the estimates reflect this; i.e., except for $i=6$ and 8, $\hat{f}_i$ is greater than $\hat{f}_i'''$.

Confidence intervals for annual survival rates are large, but the average survival rates are estimated with reasonable precision (see confidence intervals based on $\bar{\bar{S}}$, $\bar{\bar{S}}'$).

The matrices of standard normal deviates (Example 3.4d) contain few extreme values, and the goodness of fit test yields a chi-square value of 95.17 with 94 df (Example 3.4e), suggesting that agreement between model and data is good. However, both methods of assessing fit are misleading in this situation as neither provides information about how closely the H_2 model approximates the H_{2a} model. By using the H_2 model we are assuming that the survival rate for newly banded adults is not appreciably different from that for previously banded birds. As discussed above, we have no way to test this assumption, and the validity of the H_2 estimators is therefore questionable.

In-season banding of certain species is a common practice because of the convenience of banding at a time when birds are clustered together. This use of the H_2 model is suggested as an approximate method for analyzing some of the data of this sort which already exist. However, we do not thereby intend to encourage the practice of in-season banding in future programs. A decision to band during the hunting season must take into account the tradeoff between the gain in precision, due to the ease of banding large numbers of birds, and the increase in bias of the estimators. We have no idea of the magnitude of the bias likely to be incurred by using the H_2 model to describe in-season banding data.

3.6 The Model Under H_3

The assumption concerning the age-dependence of survival and recovery rates, common to H_{01}, H_{02}, H_1, and H_2, can be made more general. This leads to the hypothesis H_3, the assumptions of which are the same as those of H_2 except that survival and recovery rates are assumed to be different for three age classes, i.e., for young, subadults (birds between 1 and 2 years old), and "older birds" (birds more than 2 years old).

The experimental situation remains the same as that of previous sections in that only two age classes are recognized during banding, and we will continue to refer to these two groups as "young" and "adults." Thus every cohort of banded "adults" will contain an unknown number of subadults.

The assumptions of H_3 are:

(1) Annual survival and recovery rates are age-dependent for the first 2 years of life (i.e., are different for young, subadults, and older birds). Note that this embraces assumption 3 of model H_2 for the experimental situation we are concerned with, because of the nonidentifiability of the reporting rate λ; and

(2) annual survival and recovery rates are year-specific.

Each cohort of banded adults released contains an unknown number of subadults and under assumption 1 above, the survival and recovery rates for this mixed batch will be different from the corresponding rates which are characteristic of the groups consisting of subadults alone and all older birds. We define f_i''' and S_i''' as the rates which apply to this mixed group in the year immediately after release. Thus the parameters of H_3 are as follows:

f_i''' and S_i''' are the recovery and survival rates, respectively, in year i for the adults (i.e., subadults and older birds) banded and released in year i,

f_i' and S_i' are the recovery and survival rates, respectively, in year i for the young released in year i;

Example 3.4a

CANADA GEESE BANDED IN-SEASON AT SWAN LAKE REFUGE, 1949-57

ADULTS INPUT MATRIX

1949	828.	46.	56.	37.	32.	32.	8.	19.	12.	7.	7.	2.	3.
1950	881.	0.	35.	36.	46.	26.	17.	39.	11.	9.	7.	4.	5.
1951	379.	0.	0.	13.	17.	18.	16.	17.	10.	4.	2.	1.	1.
1952	317.	0.	0.	0.	12.	12.	9.	21.	8.	3.	3.	4.	1.
1953	358.	0.	0.	0.	0.	22.	17.	20.	13.	6.	4.	4.	4.
1954	425.	0.	0.	0.	0.	0.	36.	33.	8.	7.	10.	8.	2.
1955	833.	0.	0.	0.	0.	0.	0.	57.	40.	25.	21.	18.	15.
1956	455.	0.	0.	0.	0.	0.	0.	0.	33.	26.	25.	9.	4.
1957	433.	0.	0.	0.	0.	0.	0.	0.	0.	15.	27.	17.	8.

YOUNG INPUT MATRIX

1949	662.	59.	31.	29.	17.	19.	11.	20.	5.	5.	6.	3.	2.
1950	596.	0.	29.	36.	19.	17.	16.	9.	4.	1.	5.	0.	1.
1951	573.	0.	0.	37.	33.	19.	14.	21.	7.	4.	10.	6.	2.
1952	676.	0.	0.	0.	53.	39.	31.	37.	14.	5.	10.	3.	2.
1953	601.	0.	0.	0.	0.	51.	36.	39.	12.	7.	13.	12.	3.
1954	1160.	0.	0.	0.	0.	0.	140.	81.	22.	16.	16.	10.	6.
1955	994.	0.	0.	0.	0.	0.	0.	156.	38.	22.	25.	13.	11.
1956	850.	0.	0.	0.	0.	0.	0.	0.	105.	33.	31.	21.	16.
1957	400.	0.	0.	0.	0.	0.	0.	0.	0.	39.	27.	11.	10.

BASIC SUBTOTALS

I	RROW(I)	RCOL(I)	CROW(I)	QCOL(I)	T(I)	U(I)	W(I)	Z(I)
1	261.00	46.00	207.00	59.00	261.00	207.00	46.00	0.0
2	235.00	91.00	137.00	60.00	450.00	285.00	122.00	215.00
3	99.00	86.00	153.00	102.00	458.00	378.00	151.00	476.00
4	73.00	107.00	194.00	122.00	445.00	470.00	176.00	532.00
5	90.00	110.00	173.00	145.00	428.00	521.00	204.00	545.00
6	104.00	103.00	291.00	248.00	422.00	667.00	211.00	572.00
7	176.00	206.00	265.00	363.00	495.00	684.00	413.00	587.00
8	97.00	135.00	206.00	207.00	386.00	527.00	237.00	501.00
9	67.00	102.00	87.00	132.00	318.00	407.00	195.00	470.00
					216.00	275.00		443.00

THE HYPOTHESIS H2. (SEE BROWNIE AND ROBSON, 1974. CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

ASSUMPTIONS: (1) ANNUAL SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC.

(2) YOUNG BIRDS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES FROM THOSE OF ADULTS.

(3) IN ANY YEAR, THE REPORTING RATE FOR NEW RELEASES IS DIFFERENT FROM THAT FOR SURVIVORS OF PREVIOUSLY BANDED COHORTS, AND HENCE THE CORRESPONDING RECOVERY RATES ARE DIFFERENT.

H2 IS AN EXTENSION OF H1 IN THAT THE FIRST YEAR ADULT RECOVERY RATE IN YEAR I IS DIFFERENT FROM THE RECOVERY RATE IN YEAR I OF PREVIOUSLY BANDED ADULTS. (THE SOLICITING OF BANDS FROM HUNTERS BY CONSERVATION OFFICERS NEAR BANDING SITES MAY GIVE RISE TO THIS SITUATION).

PARAMETERS:

F'''(I)= BAND RECOVERY RATE IN YEAR I FOR ADULTS BANDED IN YEAR I.
F(I) = BAND RECOVERY RATE IN YEAR I FOR SURVIVORS OF COHORTS BANDED BEFORE YEAR I.
S(I) = SURVIVAL RATE FOR ADULTS IN YEAR I.
F'(I) = BAND RECOVERY RATE FOR YOUNG IN YEAR I.
S'(I) = SURVIVAL RATE FOR YOUNG IN YEAR I.

STRUCTURE OF THE MODEL UNDER H2 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS

N(1)F'''(1)	N(1)S(1)F(2)	N(1)S(1)S(2)F(3)	N(1)S(1)S(2)S(3)F(4)
	N(2)F'''(2)	N(2)S(2)F(3)	N(2)S(2)S(3)F(4)
		N(3)F'''(3)	N(3)S(3)F(4)

BANDED AS YOUNG

M(1)F'(1)	M(1)S'(1)F(2)	M(1)S'(1)S(2)F(3)	M(1)S'(1)S(2)S(3)F(4)
	M(2)F'(2)	M(2)S'(2)F(3)	M(2)S'(2)S(3)F(4
		M(3)F'(3)	M(3)S'(3)F(4)

Example 3.4b

ESTIMATES UNDER H2

I	YR	F(I) ESTIMATE	F(I) STANDARD ERROR	F(I) 95% CONFIDENCE INTERVAL	S(I) ESTIMATE	S(I) STANDARD ERROR	S(I) 95% CONFIDENCE INTERVAL
1	1949				0.8663	0.0783	0.7128 - 1.0199
2	1950	0.0716	0.0099	0.0522 - 0.0909	0.7573	0.0876	0.5855 - 0.9290
3	1951	0.0702	0.0095	0.0515 - 0.0889	0.8693	0.1311	0.6123 - 1.1263
4	1952	0.0652	0.0095	0.0465 - 0.0839	0.7356	0.1179	0.5045 - 0.9666
5	1953	0.0686	0.0096	0.0499 - 0.0873	0.8770	0.1380	0.6066 - 1.1474
6	1954	0.0539	0.0076	0.0390 - 0.0689	0.5756	0.0830	0.4128 - 0.7384
7	1955	0.1331	0.0150	0.1038 - 0.1625	0.6670	0.0977	0.4755 - 0.8585
8	1956	0.0707	0.0102	0.0507 - 0.0907	0.7887	0.1392	0.5159 - 1.0616
9	1957	0.0553	0.0087	0.0381 - 0.0724			

	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$	= 0.0736	0.0036	0.0665 - 0.0806
$\bar{S}$	= 0.7671	0.0170	0.7338 - 0.8003

I	YR	F'(I) ESTIMATE	F'(I) STANDARD ERROR	F'(I) 95% CONFIDENCE INTERVAL	S'(I) ESTIMATE	S'(I) STANDARD ERROR	S'(I) 95% CONFIDENCE INTERVAL
1	1949	0.0891	0.0111	0.0674 - 0.1108	0.7459	0.0745	0.5999 - 0.8919
2	1950	0.0487	0.0088	0.0314 - 0.0659	0.6045	0.0790	0.4495 - 0.7594
3	1951	0.0646	0.0103	0.0444 - 0.0847	0.7755	0.1114	0.5572 - 0.9939
4	1952	0.0784	0.0103	0.0581 - 0.0987	0.7973	0.1071	0.5873 - 1.0073
5	1953	0.0849	0.0114	0.0626 - 0.1071	0.9372	0.1304	0.6816 - 1.1929
6	1954	0.1207	0.0096	0.1019 - 0.1394	0.4683	0.0559	0.3588 - 0.5778
7	1955	0.1569	0.0115	0.1343 - 0.1796	0.5120	0.0767	0.3618 - 0.6623
8	1956	0.1235	0.0113	0.1014 - 0.1457	0.6663	0.1084	0.4539 - 0.8787
9	1957	0.0975	0.0148	0.0684 - 0.1266			

	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$	= 0.0960	0.0037	0.0888 - 0.1033
$\bar{S}'$	= 0.6884	0.0339	0.6220 - 0.7548

I	YR	F''(I) ESTIMATE	F''(I) STANDARD ERROR	F''(I) 95% CONFIDENCE INTERVAL	SK...SK+I-1FK+I ESTIMATE	SK...SK+I-1FK+I STANDARD ERROR	SK...SK+I-1FK+I 95% CONFIDENCE INTERVAL
1	1949	0.0556	0.0080	0.0400 - 0.0712	0.0609	0.0084	0.0445 - 0.0773
2	1950	0.0397	0.0066	0.0268 - 0.0526	0.0357	0.0053	0.0254 - 0.0460
3	1951	0.0343	0.0093	0.0160 - 0.0526	0.0235	0.0037	0.0162 - 0.0308
4	1952	0.0379	0.0107	0.0168 - 0.0589			
5	1953	0.0615	0.0127	0.0366 - 0.0863			
6	1954	0.0847	0.0135	0.0582 - 0.1112			
7	1955	0.0684	0.0087	0.0513 - 0.0856			
8	1956	0.0725	0.0122	0.0487 - 0.0964			
9	1957	0.0346	0.0088	0.0174 - 0.0519			

	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}''$	= 0.0544	0.0034	0.0476 - 0.0611

Example 3.4c

CANADA GEESE BANDED IN-SEASON AT SWAN LAKE REFUGE, 1949-57

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H2

I	YR	COVAR(F'''(I),F(I))	CORR(F'''(I),F(I))	COVAR(F'''(I),S(I))	CORR(F'''(I),S(I))
1	1949			-0.000058127	-0.093211773
2	1950	-0.000003227	-0.049737690	-0.000034147	-0.059233279
3	1951	-0.000006354	-0.071217249	-0.000078674	-0.064181501
4	1952	-0.000007786	-0.076151510	-0.000087838	-0.069523514
5	1953	-0.000011774	-0.097117528	-0.000150540	-0.085960804
6	1954	-0.000010753	-0.104359439	-0.000114719	-0.102271894
7	1955	-0.000010936	-0.083557977	-0.000054794	-0.064110916
8	1956	-0.000011266	-0.090916909	-0.000125723	-0.074273399
9	1957	-0.000004423	-0.057536717		

I	YR	COVAR(F'''(I+1),S(I))	CORR(F'''(I+1),S(I))	COVAR(F'''(I+1),S'(I))	CORR(F'''(I+1),S'(I))
1	1949	0.000039066	0.075782919	0.000033635	0.068597296
2	1950	0.000068534	0.083678970	0.000054705	0.074029646
3	1951	0.000103807	0.073860008	0.000092613	0.077550584
4	1952	0.000126264	0.084395831	0.000136861	0.100655835
5	1953	0.000174791	0.093795504	0.000186800	0.106022676
6	1954	0.000047282	0.065081605	0.000038468	0.078730115
7	1955	0.000106326	0.089504179	0.000081617	0.087555889
8	1956	0.000063101	0.051576477	0.000053305	0.055973691

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(S(I),F(I+1))	CORR(S(I),F(I+1))
1	1949			-0.000239601	-0.310224790
2	1950	0.000209433	0.242477212	-0.000477940	-0.571621450
3	1951	0.000548653	0.438431588	-0.000750385	-0.599952081
4	1952	0.000634953	0.564727629	-0.000601028	-0.533856266
5	1953	0.000716583	0.543755784	-0.000584460	-0.555291211
6	1954	0.000383594	0.605472811	-0.000551958	-0.444205843
7	1955	0.000639652	0.437580896	-0.000633005	-0.635745482
8	1956	0.000748481	0.527561525	-0.000737848	-0.605903168

I	YR	COVAR(F(I+1),S'(I))	CORR(F(I+1),S'(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1949	-0.000206292	-0.280809760	-0.002535504	-0.369450782
2	1950	-0.000381502	-0.505705398	-0.005917452	-0.515149064
3	1951	-0.000669466	-0.629930166	-0.008465208	-0.547734082
4	1952	-0.000651472	-0.636710910	-0.007684615	-0.472527400
5	1953	-0.000624618	-0.627678792	-0.006235566	-0.544182876
6	1954	-0.000449060	-0.537362166	-0.002765458	-0.340822761
7	1955	-0.000485898	-0.621906553	-0.007064112	-0.519363711
8	1956	-0.000623306	-0.657560223		

I	YR	COVAR(S(I),S'(I))	CORR(S(I),S'(I))	COVAR(S(I+1),S'(I))	CORR(S(I+1),S'(I))
1	1949	0.003058611	0.523986295	-0.002183030	-0.334420054
2	1950	0.004357167	0.629215732	-0.004723437	-0.455745538
3	1951	0.009277858	0.635106565	-0.007552341	-0.575102865
4	1952	0.007295091	0.577755662	-0.008329593	-0.563566130
5	1953	0.010552902	0.586329501	-0.006664004	-0.615122417
6	1954	0.002281828	0.491917489	-0.002249913	-0.412297956
7	1955	0.004867200	0.649801508	-0.005422451	-0.508058438
8	1956	0.009315956	0.617527848		

I	YR	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))
1	1949	-0.000100419	-0.121698325
2	1950	-0.000049348	-0.070839992
3	1951	-0.000087398	-0.076403828
4	1952	-0.000092471	-0.083494728
5	1953	-0.000132335	-0.089244904
6	1954	-0.000048722	-0.091199354
7	1955	-0.000080842	-0.091398080
8	1956	-0.000096829	-0.079173023

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

Example 3.4d

CANADA GEESE BANDED IN-SEASON AT SWAN LAKE REFUGE, 1949-57

MATRIX OF DATA VALUES -- ADULTS

1	46.00	56.00	37.00	32.00	32.00	8.00	19.00	12.00	7.00	7.00	2.00	3.00
2	0.0	35.00	36.00	46.00	26.00	17.00	39.00	11.00	9.00	7.00	4.00	5.00
3	0.0	0.0	13.00	17.00	18.00	16.00	17.00	10.00	4.00	2.00	1.00	1.00
4	0.0	0.0	0.0	12.00	12.00	9.00	21.00	8.00	3.00	3.00	4.00	1.00
5	0.0	0.0	0.0	0.0	22.00	17.00	20.00	13.00	6.00	4.00	4.00	4.00
6	0.0	0.0	0.0	0.0	0.0	36.00	33.00	8.00	7.00	10.00	8.00	2.00
7	0.0	0.0	0.0	0.0	0.0	0.0	57.00	40.00	25.00	21.00	18.00	15.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.00	26.00	25.00	9.00	4.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15.00	27.00	17.00	8.00

MATRIX OF EXPECTED VALUES -- ADULTS

1	46.00	51.53	38.63	31.60	24.74	17.27	24.71	8.87	5.56	6.13	3.59	2.36
2	0.0	35.00	47.26	38.66	30.27	21.14	30.24	10.85	6.81	7.50	4.40	2.89
3	0.0	0.0	13.00	21.77	17.04	11.90	17.02	6.11	3.83	4.22	2.48	1.63
4	0.0	0.0	0.0	12.00	16.18	11.30	16.17	5.80	3.64	4.01	2.35	1.55
5	0.0	0.0	0.0	0.0	22.00	17.15	24.53	8.80	5.52	6.08	3.57	2.35
6	0.0	0.0	0.0	0.0	0.0	36.00	32.80	11.77	7.38	8.14	4.77	3.14
7	0.0	0.0	0.0	0.0	0.0	0.0	57.00	39.80	24.97	27.50	16.13	10.60
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	33.00	20.18	22.22	13.03	8.57
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15.00	26.37	15.46	10.17

MATRIX OF STANDARD NORMAL DEVIATES -- ADULTS

1	0.00	0.64	-0.27	0.07	1.48	-2.26	-1.17	1.06	0.61	0.35	-0.84	0.41
2	0.0	0.00	-1.68	1.21	-0.79	-0.91	1.62	0.05	0.84	-0.18	-0.19	1.24
3	0.0	0.0	0.00	-1.05	0.24	1.21	-0.01	1.59	0.09	-1.09	-0.94	-0.49
4	0.0	0.0	0.0	0.00	-1.07	-0.70	1.23	0.92	-0.34	-0.51	1.08	-0.44
5	0.0	0.0	0.0	0.0	0.00	-0.04	-0.95	1.43	0.20	-0.85	0.23	1.08
6	0.0	0.0	0.0	0.0	0.0	0.00	0.04	-1.11	-0.14	0.66	1.49	-0.64
7	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.03	0.01	-1.26	0.47	1.36
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	1.33	0.60	-1.13	-1.58
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00	0.13	0.40	-0.69

MATRIX OF DATA VALUES -- YOUNG

1	59.00	31.00	29.00	17.00	19.00	11.00	20.00	5.00	5.00	6.00	3.00	2.00
2	0.0	29.00	36.00	19.00	17.00	16.00	9.00	4.00	1.00	5.00	0.0	1.00
3	0.0	0.0	37.00	33.00	19.00	14.00	21.00	7.00	4.00	10.00	6.00	2.00
4	0.0	0.0	0.0	53.00	39.00	31.00	37.00	14.00	5.00	10.00	3.00	2.00
5	0.0	0.0	0.0	0.0	51.00	36.00	39.00	12.00	7.00	13.00	12.00	3.00
6	0.0	0.0	0.0	0.0	0.0	140.00	81.00	22.00	16.00	16.00	10.00	6.00
7	0.0	0.0	0.0	0.0	0.0	0.0	156.00	38.00	22.00	25.00	13.00	11.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	105.00	33.00	31.00	21.00	16.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	39.00	27.00	11.00	10.00

MATRIX OF EXPECTED VALUES -- YOUNG

1	59.00	35.47	26.59	21.75	17.03	11.89	17.01	6.10	3.83	4.22	2.47	1.63
2	0.0	29.00	25.52	20.87	16.34	11.41	16.33	5.86	3.68	4.05	2.37	1.56
3	0.0	0.0	37.00	29.36	22.99	16.05	22.96	8.24	5.17	5.70	3.34	2.20
4	0.0	0.0	0.0	53.00	37.41	26.12	37.37	13.41	8.41	9.27	5.43	3.57
5	0.0	0.0	0.0	0.0	51.00	30.76	44.01	15.79	9.91	10.92	6.40	4.21
6	0.0	0.0	0.0	0.0	0.0	140.00	72.84	26.14	16.40	18.06	10.59	6.96
7	0.0	0.0	0.0	0.0	0.0	0.0	156.00	36.45	22.87	25.19	14.77	9.71
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	105.00	31.84	35.07	20.57	13.52
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	39.00	24.34	14.27	9.38

MATRIX OF STANDARD NORMAL DEVIATES -- YOUNG

1	0.0	-0.77	0.48	-1.04	0.48	-0.26	0.73	-0.45	0.60	0.87	0.34	0.29
2	0.0	0.0	2.12	-0.42	0.16	1.37	-1.84	-0.77	-1.40	0.47	-1.54	-0.45
3	0.0	0.0	0.0	0.69	-0.85	-0.52	-0.42	-0.44	-0.52	1.81	1.46	-0.13
4	0.0	0.0	0.0	0.0	0.27	0.97	-0.06	0.16	-1.18	0.24	-1.05	-0.83
5	0.0	0.0	0.0	0.0	0.0	0.97	-0.78	-0.97	-0.93	0.64	2.23	-0.59
6	0.0	0.0	0.0	0.0	0.0	0.0	0.99	-0.82	-0.10	-0.49	-0.18	-0.37
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.26	-0.18	-0.04	-0.46	0.42
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.21	-0.70	0.10	0.68
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.56	-0.88	0.20

Example 3.4e

CANADA GEESE BANDED IN-SEASON AT SWAN LAKE REFUGE, 1949-57

CHI-SQUARE TEST OF H1 VS H2

	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
I= 2	35	200	
	87	276	7.232
I= 3	13	86	
	138	446	5.418
I= 4	12	61	
	164	484	2.798
I= 5	22	68	
	182	504	0.179
I= 6	36	68	
	175	519	4.108
I= 7	57	119	
	356	382	14.418
I= 8	33	64	
	204	406	0.013
I= 9	15	52	
	180	351	2.358

TOTAL CHI-SQUARE WITH 8 DEGREES OF FREEDOM = 36.523

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 36.52 = 0.00001

THIS TEST OF THE HYPOTHESIS H1 AGAINST THE HYPOTHESIS H2 TESTS THE ASSUMPTION THAT RECOVERY RATES FOR NEWLY RELEASED ADULTS ARE THE SAME AS FOR SURVIVORS OF PREVIOUSLY BANDED COHORTS.

CHI-SQUARE TEST OF H2 VS H3

TOTAL CHI-SQUARE WITH 9 DEGREES OF FREEDOM = 13.532

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 13.53 = 0.13999

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H0

TOTAL CHI-SQUARE 252.54 WITH120 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 252.54 = 0.0

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H1

TOTAL CHI-SQUARE 131.69 WITH102 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 131.69 = 0.02551

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H2

TOTAL CHI-SQUARE 95.17 WITH 94 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 95.17 = 0.44686

FROM THE MODELS ABOVE, ONE SHOULD CHOOSE THE SIMPLEST MODEL (FEWEST PARAMETERS) THAT ADEQUATELY DESCRIBES THE DATA. ADEQUACY MAY BE JUDGED BY EXAMINING THE RESULTS OF (1) THE GOODNESS OF FIT TESTS, AND (2) THE TESTS BETWEEN SPECIFIC MODELS. FREQUENTLY, H02 OR H1 IS ADEQUATE.

f_i'' and S_i'' are the recovery and survival rates, respectively, in year i for the subadults (i.e., survivors of young released in year $i-1$); and

f_i and S_i are the recovery and survival rates, respectively, in year i for birds over 2 years old.

The structure of the model under H_3 is expressed in Table 3.7 by the expected numbers of band recoveries in terms of $N_i, M_i, f_i, f_i', f_i'', f_i''', S_i, S_i', S_i''$ and S_i'''.

Under this model the only parameters which are separately estimable are $f_i', f_i''', i=1,\ldots,k;$ thus estimation under this model is of little interest and is omitted here. If assumption 1 of H_3 is true, then the H_1 and H_2 estimates will be biased, and it is therefore of interest to attempt to determine whether this assumption is necessary. To this end, the data can be used to compute a goodness of fit test to the H_3 model and a test to compare H_2 with the more general H_3 (see Section 3.7).

Table 3.7. *Expected numbers of band recoveries under H_3 for a banding study with $k=3$, $\ell=5$, $s=0$.*

Year banded	Number banded	Year of recovery 1	2	3	4	5
		Birds banded and released as adults (i.e., as subadults or older)				
1	N_1	N_1f_1'''	$N_1S_1'''f_2$	$N_1S_1'''S_2f_3$	$N_1S_1'''S_2S_3f_4$	$N_1S_1'''S_2S_3S_4f_5$
2	N_2		N_2f_2'''	$N_2S_2'''f_3$	$N_2S_2'''S_3f_4$	$N_2S_2'''S_3S_4f_5$
3	N_3			N_3f_3'''	$N_3S_3'''f_4$	$N_3S_3'''S_4f_5$
		Birds banded and released as young				
1	M_1	M_1f_1'	$M_1S_1'f_2''$	$M_1S_1'S_2''f_3$	$M_1S_1'S_2''S_3f_4$	$M_1S_1'S_2''S_3S_4f_5$
2	M_2		M_2f_2'	$M_2S_2'f_3''$	$M_2S_2'S_3''f_4$	$M_2S_2'S_3''S_4f_5$
3	M_3			M_3f_3'	$M_3S_3'f_4''$	$M_3S_3'S_4''f_5$

Goodness of Fit Test

A goodness of fit test to H_3 is computed by the FORTRAN program BROWNIE. The test statistic is obtained by summing the chi-square values from a series of contingency tables, the rows of which are obtained from rows of the recovery matrices in the manner described in Brownie (1973). Again, because observations in the upper right portion of the recovery matrices are frequently 0 or 1, pooling is usually necessary to justify the chi-square approximation, and the FORTRAN program does this using an algorithm adapted from a method described in Robson (1971c). The program prints the contingency tables, as they appear after pooling, together with the chi-square value and associated degrees of freedom (see Example 3.5p). Pooling, if necessary, is accomplished by combining entries in the same row. Occasionally, if the data are very scant, pooling may result in a single column and no chi-square value can be computed. In this situation, the single column is printed together with the message "NO CHI-SQUARE COMPUTED."

Summing the chi-square values and (separately) the degrees of freedom from the contingency tables gives the test statistic (printed as the "TOTAL CHI-SQUARE") and its degrees of freedom. In Example 3.5p, the goodness of fit test to H_3 for the mallard data of Table 3.2 yields a total chi-square of 34.07 with 34 df, and under H_3, the probability of a value at least this large is 0.46. Thus there is no reason to reject the H_3 model.

3.7 Testing Between Models

The models corresponding to H_{01}, H_{02}, H_1, H_2, and H_3 become increasingly more complex as the assumptions of the hypotheses become more general. For a given data set, if we use estimators from too simple a model (a model with too few parameters) they will be biased, but if we use too general a model (a model with more parameters than are necessary) the estimators will not be efficient (i.e., their precision will be unnecessarily low). In that we want to choose the simplest model that adequately describes the data, we must examine the results of the goodness of fit tests and of tests which compare one model with another as described below.

Each of the tests in this section compares a given model with a more general alternative, and "large" values of the test statistic indicate that the simpler model should be rejected in favor of the more general alternative.

Likelihood Ratio Test of H_{01} vs. H_{02}

The simplest model introduced so far in this chapter is the H_{01} model which is compared with the H_{02} model by means of a likelihood ratio test. For large N_i and M_i the test statistic is chi-square distributed with $k+\ell-2$ degrees of freedom if H_{01} is true. In the FORTRAN program, this test is printed out after the goodness of fit test to H_{02} and before the H_1 estimates (see Example 3.5e).

As mentioned in Section 3.4 the H_{01} model is apparently too simple for most data sets, and the test usually results in a "large" chi-square value and a clearcut rejection of the H_{01} model in favor of the H_{02} model. The assumption that adult and young recovery rates are constant from year to year is usually too restrictive.

For the blue-winged teal data of Example 3.3 the chi-square value for this test is 72.72 with 17 df, and the probability of a value this large, if H_{01} is true, is virtually zero.

For the mallard data of Example 3.5e, the chi-square value is 77.35 with 16 df and again H_{01} is rejected in favor of H_{02}.

Likelihood Ratio Test of H_{02} vs. H_1

The model under H_{02} is compared with the model under H_1 by means of a likelihood ratio test. If H_{02} is true, for large N_i and M_i, the test statistic is chi-square with degrees of freedom given by

$$2k-4 \text{ if } s=0\ (\ell=k)$$
$$2k-3 \text{ if } s>0\ (\ell>k).$$

This test is of interest because, as mentioned earlier, results obtained by the analysis of a large number of data sets indicate that these two models are frequently the most useful, with respect to the criteria above. The test is also of interest because of the relationship of H_{02} to the "hypothesis of compensatory natural mortality." Thus, the test of H_{02} against H_1 essentially tests the assumption that, in the presence of variable recovery rates, adult and young survival rates are constant from year to year. Rejection of the H_{02} assumption of a constant survival rate is necessary before it is possible to say that natural mortality does not compensate fully for variations in hunting losses. The proper formulation of, and testing for, additivity vs. compensatory mortality is dealt with in Anderson and Burnham (1976).

Unfortunately for values of N_i and M_i, usually encountered in practice, the power of this test is low, i.e., the test is not very likely to result in rejection of H_{02} when differences between the annual survival rates are small. The test is computed in the FORTRAN program and the result is printed following a series of tests between H_0, H_1, H_2, and H_3 (where H_0 is defined below) as indicated in Examples 3.5n and 3.5o.

For the blue-winged teal data of Example 3.2c, the test results in a chi-square value of 15.14 with 11 df (inadvertently not photographed), and the H_{02} model is not rejected. In view of this result, the low recovery rates and hence low precision of the estimators, the H_{02} model is probably the most appropriate model for this particular set of data.

For the mallard data in Example 3.5, this test is significant at approximately the 2% level (chi-square value = 26.94 with 14 df, cf. Example 3.5o), and the H_1 model is preferred to the H_{02} model for these data. Having rejected the model under H_{02}, we must compare the model under H_1 with the still more general models under H_2 and H_3, as described below, before we can determine which is the most appropriate model for these data.

Testing Between H_0, H_1, H_2, and H_3

Common to all the models introduced so far in this chapter is the assumption that survival and recovery rates are different for young and adults, so that testing between these models will not provide a test of this assumption. We therefore define the H_0 model, the assumptions of which are that survival, hunting, reporting, and hence recovery, rates are year-specific but independent of age. The model under H_0 is equivalent to assuming Model 1 of Chapter 2 applies to the pooled adult and young recovery data. Like H_{01} and H_{02}, this leads to a model which is simpler than the H_1 model, but which is not comparable with the H_{01} and H_{02} models. The test of H_0 against H_1, tests the assumption that annual survival and recovery rates are independent of age. We also test H_1 against the more general H_2, and H_2 against the still more general H_3.

These three tests are not likelihood ratio tests; rather they are analogous to the goodness of fit tests to the models under H_1, H_2, and H_3 referred to above. The derivation of both these tests between models and the goodness of fit tests to these three models is outlined in Brownie and Robson (1976).

H_0 *vs.* H_1

The test of the null hypothesis that the H_0 model fits the data against the alternative H_1 model (or H_0 vs. H_1) is based on a statistic obtained by summing the single degree of freedom chi-square statistics from each of the 2x2 contingency tables given by

$R_{i\cdot}$	$N_i - R_{i\cdot}$	N_i
$Q_{i\cdot}$	$M_i - Q_{i\cdot}$	M_i
		$N_i + M_i$

$, i = 1, \ldots, k$, and

W_i	Z_{i+1}	$T_i + U_i - Q_{i\cdot}$
Q_{ii}	$Q_{i\cdot} - Q_{ii}$	$Q_{i\cdot}$
		$T_i + U_i$

$$, i = \begin{cases} 1, \ldots, k-1 & \text{if } s = 0 \\ 1, \ldots, k & \text{if } s > 0, \end{cases}$$

where $Z_{i+1} = T_i + U_i - Q_{i\cdot} - W_i$, $i = 1, \ldots, k$.

The contingency tables and chi-square values are printed together with the total chi-square and degrees of freedom as in Example 3.5n. For the mallard data of Table 3.2 as shown in Example 3.5n, the contingency tables for $i = 2$ are

$R_{2\cdot}$	$N_2 - R_{2\cdot}$
$Q_{2\cdot}$	$M_2 - Q_{2\cdot}$

and

W_2	$T_2 + U_2 - Q_{2\cdot} - W_2$
Q_{22}	$Q_{2\cdot} - Q_{22}$

or

131	518
168	534

and

106	144
103	65

with chi-square values 2.747 and 14.371, respectively. The total chi-square for these data is 129.853 with 17 df, and if the model under H_0 is true, a value this extreme is highly unlikely. In Example 3.5, the model under H_0 is rejected, and we infer that survival and recovery rates are different for young and adults.

Note that although the test is based on the total chi-square value, information concerning particular years of the experiment may be obtained by examining the chi-square values from the contingency tables individually. This may be of more value in testing H_1 against H_2 (see below), but we indicate here how each of the contingency tables above is associated with a particular year of the experiment.

The first series of contingency tables tests the equality of the ratios $R_{i\cdot}/N_i$ and $Q_{i\cdot}/M_i$, where $R_{i\cdot}/N_i$ is the proportion of the adults released in year i which are subsequently recovered, and $Q_{i\cdot}/M_i$ is the proportion of the young released in year i which are subsequently recovered.

Clearly, if survival and recovery rates are the same for adults and young in subsequent years (i.e., $f_j' = f_j$, $S_j = S_j'$, $j = 1, \ldots, \ell$), these two fractions will have the same average or expected value.

The second series of tables tests the equality of $W_i/(T_i + U_i - Q_{i\cdot})$ and $Q_{ii}/Q_{i\cdot}$, where the former ratio is the proportion of the total recoveries from banded *adults* alive at the start of year i (including survivors of previously released young), which were recovered in year i, and $Q_{ii}/Q_{i\cdot}$ is the proportion of the total recoveries from the *young* released in year i, which were recovered in year i. If $f_i = f_i'$ and $S_i = S_i'$, these fractions should not differ significantly.

For example, the contingency table

W_i	Z_{i+1}
Q_{ii}	$Q_{i\cdot} - Q_{ii}$

for $i=4$ (year 1966) of Example 3.5n gives a large chi-square value of 42.149. Examination of the H_1 estimates (Example 3.5f) suggests this may be due to an unusually high estimated adult survival rate ($\tilde{S}_4=0.8051$) in 1966-67, reflected in the fraction $Z_5/(T_4+U_4-Q_{4\cdot})=316/481$; and also to a high estimated recovery rate for young $\hat{f}_4'=0.1274$, reflected in the fraction $Q_{44}/Q_{4\cdot}=153/259$.

The analysis of a large number of data sets with program BROWNIE indicates that this test is always extremely significant, and hence that there are real differences in the survival and recovery rates for young and adults of a broad range of species of waterfowl. If, however, this test did not result in rejection of H_0, the data for young and adults should be pooled and analyzed by the methods of Chapter 2.

H_1 vs. H_2

The test of the H_1 model against H_2 is based on the sum of the single degrees of freedom chi-square statistics obtained from the contingency tables.

R_{ii}	$R_{i\cdot} - R_{ii}$	$R_{i\cdot}$
$W_i - R_{ii}$	$Z_{i+1} - R_{i\cdot} + R_{ii}$	$T_i + U_i - Q_{i\cdot} - R_{i\cdot}$
		$T_i + U_i - Q_{i\cdot} = W_i + Z_{i+1}$

$$, \; i=\begin{cases}2,\ldots,k-1 & \text{if } \ell=k\\ 2,\ldots,k & \text{if } \ell>k.\end{cases}$$

This sum is printed by the FORTRAN program as the "TOTAL CHI-SQUARE" with the degrees of freedom which are $k-2$ if $\ell=k$ and $k-1$ if $\ell>k$. Each of the contingency tables and corresponding chi-square values are also printed, and as described above, the individual chi-square values may contain useful information.

The individual chi-squares test the equality of the ratios $R_{ii}/R_{i\cdot}$ and $(W_i-R_{ii})/(T_i+U_i-Q_{i\cdot}-R_{i\cdot})$, where $R_{ii}/R_{i\cdot}$ is the proportion of the total recoveries from the adults released in year i which were recovered in the year immediately after release (i.e., in year i), and $(W_i-R_{ii})/(T_i+U_i-Q_{i\cdot}-R_{i\cdot})$ is the proportion of the total recoveries from the adults alive at the start of year i, but released before year i, which were also recovered in year i. Under H_1, if $f_i'''=f_i$, these two fractions should not differ significantly.

In Example 3.5o, the test of H_1 vs. H_2 gives a total chi-square value of 14.766 with 7 df. The probability under H_1 of observing a value larger than this is 0.04 and we would usually reject H_1 in favor of H_2. However, examining the individual chi-square values we see that only one of these is significantly large, namely the value from the $i=8$ table (corresponding to the year 1970-71) which is 9.856, and which constitutes a large proportion of the total chi-square value of 14.766. Note that this corresponds to the year for which the difference between the H_2 estimates $\hat{f}_i''$ and $\hat{f}_i$ is greatest (viz $\hat{f}_8=0.0464$, $\hat{f}_8''=0.1034$). This suggests that the bias of the H_1 estimators of f_i and S_i is probably small except for this year. In view of the poor precision of the H_2 estimators the H_1 model is probably better for these data, although the test of H_1 vs. H_2 would, at first glance, suggest that H_1 is not appropriate. Before deciding that H_1 is the model to use, however, the goodness of fit tests and the test of H_2 vs. H_3 (see below) should also be examined.

Example 3.6 contains a data set for which the H_2 model does seem to be appropriate, and this test of H_1 vs. H_2 is also discussed there.

H_2 vs. H_3

This test of H_2 against the alternative H_3 is based on the sum of the single degrees of freedom chi-square statistics obtained from the contingency tables

$Q_{i-1,i}$	$Q_{i-1\cdot}-Q_{i-1,i-1}-Q_{i-1,i}$	$Q_{i-1\cdot}-Q_{i-1,i-1}$
$W_i-R_{ii}-Q_{i-1,i}$	$Z_{i+1}-R_{i\cdot}+R_{ii}-Q_{i-1,i-1}+Q_{i-1,i}$	$T_i+U_i-Q_{i\cdot}-R_{i\cdot}-Q_{i-1\cdot}+Q_{i-1,i-1}$
		$T_i+U_i-Q_{i\cdot}-R_{i\cdot}$

$$, i=\begin{cases}2,\ldots,k-1 & \text{if } \ell=k\\ 2,\ldots,k & \text{if } \ell>k\end{cases},$$

and if $\ell > k+1$, there is one additional table,

$Q_{k,k+1}$	$Q_{k\cdot}-Q_{kk}-Q_{k,k+1}$	$Q_{k\cdot}-Q_{kk}$
$R_{\cdot k+1}+Q_{\cdot k+1}-Q_{k,k+1}$	$T_{k+1}+U_{k+1}-Q_{k\cdot}+Q_{kk}-R_{\cdot k+1}-Q_{\cdot k+1}+Q_{k,k+1}$	$T_{k+1}+U_{k+1}-Q_{k\cdot}+Q_{kk}$
		$T_{k+1}+U_{k+1}$

The above tables and corresponding chi-square values are printed by the FORTRAN program together with the total chi-square value and degrees of freedom which are $k-2$, $k-1$ or k if $\ell=k$, $\ell=k+1$ or $\ell>k+1$, respectively.

The individual chi-squares are sensitive to differences between the ratios f_i''/S_i'' and f_i/S_i, where f_i'' and S_i'', as defined in Section 3.6, are rate parameters for subadults. Thus, if survival and recovery rates are different for subadults as well as for young (i.e., if $f_i''\neq f_i$, $S_i''\neq S_i$) and if $f_i''/S_i''\neq f_i/S_i$, then the test should usually result in rejection of H_2 in favor of H_3 (if banded samples N_i and M_i are sufficiently large).

The test of H_2 vs. H_3 for the data of Table 3.2, as given in Example 3.5o shows that none of the individual chi-square values nor the total chi-square value, is significantly large. Thus, there is no reason to reject H_2. This result is typical of the analysis of most data sets for a broad variety of waterfowl species, and there does not seem to be an indication that subadults have different survival and recovery rates than older birds. The H_3 model appears to be too general for these data.

An Example

Example 3.5 consists of a complete output from the FORTRAN program containing the analysis of the data of Table 3.2 and has been referred to in most of the preceding sections. Here we illustrate how the complete output is used to determine which model is the most appropriate for a given data set.

Note that the H_1 adult recovery rate estimates $\hat{f}_i$ are slightly larger than the corresponding H_2 estimates, except for $\hat{f}_8=0.0888$ which is considerably larger than the H_2 estimate of 0.0464. This is accounted for by noting that except for $i=6$, the H_2 estimates of recovery rates for newly released adults, $\hat{f}_i''$, are larger than the corresponding H_2 estimates $\hat{f}_i$, with $\hat{f}_8''$ considerably larger than $\hat{f}_8$. There is therefore some indication that recovery rates are higher for newly banded birds.

Note also that the confidence intervals based on the H_2 estimates of survival rates are large, and in some instances contain inadmissable values (i.e., values greater than 1). Comparison of the standard errors and confidence intervals for the H_1 and H_2 estimators shows that the loss in precision incurred by using the H_2 estimators is considerable. For example, under H_1, se$(\hat{f}_2)=0.0092$, se$(\tilde{S}_1)=0.1134$, se$(\tilde{S}_1')=0.0594$, while under H_2, se$(\hat{f}_2)=0.0165$, se$(\tilde{S}_1)=0.1377$, se$(\tilde{S}_1')=0.0834$. The decision to choose model H_2 over model H_1 for this data set must take into account the tradeoff between the decrease in bias versus the loss in precision associated with use of the H_2 estimators.

Note the high estimated sampling correlations between $\hat{f}_i$ and $\tilde{S}_i$, denoted by CORR(F(I),S(I)) in Example 3.5g. These correlations make it virtually impossible to use the estimates $\hat{f}_i$ and $\tilde{S}_i$ to obtain information about the relationship between parameters f and S.

The goodness of fit tests to H_0, H_1, H_2, and H_3 on Example 3.5p show that the H_0 model is inappropriate (chi-square = 182.27 with 65 df) but do not indicate a lack of fit to any of the other models. The test of H_{01} vs. H_{02} in Example 3.5e rules out H_{01}. The goodness of fit test to H_{02} (Example 3.5e) and the test of H_{02} vs. H_1 (Example 3.5o) show that H_1 is preferable to H_{02}. Comparing H_1 with the more general H_2 shows that except for 1 extreme year (or possibly 2 years), the H_1 model seems adequate. Lastly, comparing H_2 and H_3 does not indicate any reason for using H_3 instead of H_2. Thus the best model for these data (with regard to the conflicting aims of minimizing bias and maximizing precision of the estimators) is that under either H_1 or H_2. In view of the low precision of the H_2 estimators, H_1 seems preferable.

Example 3.5a

THIS OUTPUT REPRESENTS A STATISTICAL ANALYSIS OF BANDING DATA WHEN ANNUAL RECOVERIES ARE RECORDED SEPARATELY FOR BIRDS BANDED AS ADULTS AND FOR BIRDS BANDED AS YOUNG. DIFFERENT ASSUMPTIONS ABOUT ANNUAL SURVIVAL AND RECOVERY RATES GIVE RISE TO A SERIES OF HYPOTHESES AND A CORRESPONDING SERIES OF STOCHASTIC MODELS. ESTIMATES OF SURVIVAL AND RECOVERY RATES ARE COMPUTED FOR MOST MODELS AND THE ASSUMPTIONS OF THE HYPOTHESES ARE EXAMINED BY TESTS OF GOODNESS OF FIT TO THE MODELS AND BY TESTS TO DISCRIMINATE BETWEEN ALTERNATIVE MODELS.

DEFINITIONS AND NOTATION

THE MODELS ARE DEFINED IN TERMS OF TWO BASIC PARAMETERS- S, THE ANNUAL SURVIVAL RATE, AND F, THE ANNUAL BAND RECOVERY RATE. THAT IS, FOR A BIRD ALIVE AT THE START OF A YEAR, S IS THE PROBABILITY IT SURVIVES THE YEAR, AND F IS THE PROBABILITY IT IS SHOT AND ITS BAND RETURNED TO THE BIRD BANDING LABORATORY WITHIN THE YEAR. A 'YEAR' IS THE PERIOD BETWEEN SUCCESSIVE BANDINGS, OR BETWEEN ANNIVERSARIES OF THE BANDING TIME. DIFFERENT ASSUMPTIONS ARE MADE ABOUT THE YEAR-SPECIFICITY AND AGE-DEPENDENCE OF S AND F UNDER EACH HYPOTHESIS AND THE RESULTING PARAMETERS ARE DEFINED SEPARATELY FOR EACH HYPOTHESIS. IN GENERAL, AN INDEX ON S OR F DENOTES YEAR-SPECIFICITY, AND A SUPERSCRIPT ' DENOTES AGE-DEPENDENCE. FOR EXAMPLE, S(I) IS A SURVIVAL RATE FOR YEAR I, AND S'(I) IS A SURVIVAL RATE FOR YOUNG IN YEAR I.

OTHER NOTATION WHICH IS COMMON TO ALL MODELS FOLLOWS.

K	THE NUMBER OF YEARS OF BANDING
L	THE NUMBER OF YEARS OF RECOVERY
N(I)	THE NUMBER OF ADULTS BANDED AND RELEASED AT THE START OF YEAR I, I=1,...,K.
M(I)	THE NUMBER OF YOUNG BANDED AND RELEASED AT THE START OF YEAR I, I=1,...,K.
RROW(I)	ROW TOTAL OF THE RECOVERY MATRIX FOR ADULTS, OR THE TOTAL NUMBER OF RECOVERIES FROM THE ADULTS RELEASED IN YEAR I, I=1,...,K.
R(I.)	SAME AS RROW(I)
QROW(I)	ROW TOTAL OF THE RECOVERY MATRIX FOR YOUNG, OR THE TOTAL NUMBER OF RECOVERIES FROM THE YOUNG RELEASED IN YEAR I, I=1,...,K.
Q(I.)	SAME AS QROW(I)
RCOL(I)	COLUMN TOTAL OF THE RECOVERY MATRIX FOR ADULTS, I=1,...,L.
QCOL(I)	COLUMN TOTAL OF THE RECOVERY MATRIX FOR YOUNG , I=1,...,L.
Q(I,I)	NUMBER OF RECOVERIES IN YEAR I FROM THE YOUNG RELEASED IN YEAR I, I=1,...,K.
W(I)	=RCOL(I)+QCOL(I)-Q(I,I)= TOTAL NUMBER OF ADULTS RECOVERED IN YEAR I, (INCLUDING RECOVERIES FROM SURVIVORS OF YOUNG RELEASED BEFORE YEAR I), I=1,...,K
T(I)	BLOCK TOTAL OF THE RECOVERY MATRIX FOR ADULTS, OR THE TOTAL NUMBER OF RECOVERIES IN YEARS I TO L INCLUSIVE, FROM ALL THE ADULTS RELEASED IN YEARS 1 TO I INCLUSIVE.
U(I)	BLOCK TOTAL OF THE RECOVERY MATRIX FOR YOUNG , OR THE TOTAL NUMBER OF RECOVERIES IN YEARS I TO L INCLUSIVE, FROM ALL THE YOUNG RELEASED IN YEARS 1 TO I INCLUSIVE.
Z(I)	=T(I)+U(I)-RROW(I)-QROW(I)-QROW(I-1)+Q(I-1,I-1), A TOTAL INVOLVING BLOCKS FROM BOTH DATA ARRAYS, I=2,...,K

Example 3.5b

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

ADULTS INPUT MATRIX

1963	231.	10.	13.	6.	1.	1.	3.	1.	2.	0.
1964	649.	0.	58.	21.	16.	15.	13.	6.	1.	1.
1965	885.	0.	0.	54.	39.	23.	18.	11.	10.	6.
1966	550.	0.	0.	0.	44.	21.	22.	9.	9.	3.
1967	943.	0.	0.	0.	0.	55.	39.	23.	11.	12.
1968	1077.	0.	0.	0.	0.	0.	66.	46.	29.	18.
1969	1250.	0.	0.	0.	0.	0.	0.	101.	59.	30.
1970	938.	0.	0.	0.	0.	0.	0.	0.	97.	22.
1971	312.	0.	0.	0.	0.	0.	0.	0.	0.	21.

YOUNG INPUT MATRIX

1963	962.	83.	35.	18.	16.	6.	8.	5.	3.	1.
1964	702.	0.	103.	21.	13.	11.	8.	6.	6.	0.
1965	1132.	0.	0.	82.	36.	26.	24.	15.	18.	4.
1966	1201.	0.	0.	0.	153.	39.	22.	21.	16.	8.
1967	1159.	0.	0.	0.	0.	109.	38.	31.	15.	1.
1968	1155.	0.	0.	0.	0.	0.	113.	64.	29.	22.
1969	1131.	0.	0.	0.	0.	0.	0.	124.	45.	22.
1970	906.	0.	0.	0.	0.	0.	0.	0.	95.	25.
1971	353.	0.	0.	0.	0.	0.	0.	0.	0.	38.

BASIC SUBTOTALS

I	RROW(I)	RCOL(I)	CROW(I)	QCOL(I)	T(I)	U(I)	W(I)	Z(I)
1	37.00	10.00	175.00	83.00	37.00	175.00	10.00	0.0
2	131.00	71.00	168.00	138.00	158.00	260.00	106.00	27.00
3	161.00	81.00	205.00	121.00	248.00	327.00	120.00	144.00
4	108.00	100.00	259.00	218.00	275.00	465.00	165.00	250.00
5	140.00	115.00	194.00	191.00	315.00	441.00	197.00	316.00
6	159.00	161.00	228.00	213.00	359.00	478.00	261.00	365.00
7	190.00	197.00	191.00	266.00	388.00	456.00	339.00	348.00
8	119.00	218.00	120.00	227.00	310.00	310.00	350.00	314.00
9	21.00	113.00	38.00	121.00	113.00	121.00	196.00	150.00

THE HYPOTHESIS H01

ASSUMPTIONS: (1) YOUNG AND ADULTS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES

(2) OTHERWISE, SURVIVAL AND RECOVERY RATES ARE CONSTANT FROM YEAR TO YEAR

PARAMETERS:

S = CONSTANT ANNUAL SURVIVAL RATE FOR ADULTS
F = CONSTANT BAND RECOVERY RATE FOR ADULTS
S' = CONSTANT ANNUAL SURVIVAL RATE FOR YOUNG
F' = CONSTANT BAND RECOVERY RATE FOR YOUNG

STRUCTURE OF THE MODEL UNDER H01 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS				BANDED AS YOUNG			
N(1)F	N(1)SF	N(1)SSF	N(1)SSSF	M(1)F'	M(1)S'F	M(1)S'SF	M(1)S'SSF
	N(2)F	N(2)SF	N(2)SSF		M(2)F'	M(2)S'F	M(2)S'SF
		N(3)F	N(3)SF			M(3)F'	M(3)S'F

ESTIMATES UNDER H01

F RECOVERY RATE FOR ADULTS

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.0677	0.0023	0.0631 - 0.0722

F' RECOVERY RATE FOR YOUNG

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.1030	0.0033	0.0966 - 0.1093

S SURVIVAL RATE FOR ADULTS

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.6489	0.0115	0.6264 - 0.6715

S' SURVIVAL RATE FOR YOUNG

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.5403	0.0250	0.4913 - 0.5892

ESTIMATED COVARIANCES AND CORRELATIONS UNDER H01

COV(F,F') -0.00000000	CORR(F,F') -0.0000	COV(F,S') -0.00002921	CORR(F,S') -0.5032
COV(F,S) -0.00001545	CORR(F,S) -0.5776	COV(S,F') 0.00000000	CORR(S,F') 0.0000
COV(S,S') 0.00000370	CORR(S,S') 0.0129	COV(F',S') -0.00000636	CORR(F',S') -0.0784

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

NUMBER OF ITERATIONS COMPLETED = 4

Example 3.5c

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

THE HYPOTHESIS H02

ASSUMPTIONS: (1) YOUNG AND ADULTS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES

(2) SURVIVAL RATES ARE OTHERWISE CONSTANT FROM YEAR TO YEAR

(3) RECOVERY RATES ARE YEAR-SPECIFIC

PARAMETERS:

S = CONSTANT ANNUAL SURVIVAL RATE FOR ADULTS
F(I) = BAND RECOVERY RATE IN YEAR I FOR ADULTS
S' = CONSTANT ANNUAL SURVIVAL RATE FOR YOUNG
F'(I) = BAND RECOVERY RATE IN YEAR I FOR YOUNG

STRUCTURE OF THE MODEL UNDER H02 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS				BANDED AS YOUNG			
N(1)F(1)	N(1)SF(2)	N(1)SSF(3)	N(1)SSSF(4)	M(1)F'(1)	M(1)S'F(2)	M(1)S'SF(3)	M(1)S'SSF(4)
	N(2)F(2)	N(2)SF(3)	N(2)SSF(4)		M(2)F'(2)	M(2)S'F(3)	M(2)S'SF(4)
		N(3)F(3)	N(3)SF(4)			M(3)F'(3)	M(3)S'F(4)

ESTIMATES UNDER H02

	F(I) RECOVERY RATE FOR ADULTS			F'(I) RECOVERY RATE FOR YOUNG		
I YEAR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1 1963	0.0429	0.0133	0.0169 - 0.0690	0.0855	0.0090	0.0679 - 0.1031
2 1964	0.0804	0.0077	0.0654 - 0.0955	0.1468	0.0133	0.1208 - 0.1728
3 1965	0.0569	0.0052	0.0467 - 0.0670	0.0736	0.0077	0.0585 - 0.0888
4 1966	0.0644	0.0051	0.0543 - 0.0745	0.1270	0.0096	0.1083 - 0.1458
5 1967	0.0600	0.0045	0.0512 - 0.0687	0.0891	0.0082	0.0730 - 0.1051
6 1968	0.0669	0.0044	0.0581 - 0.0756	0.0998	0.0088	0.0825 - 0.1170
7 1969	0.0769	0.0046	0.0680 - 0.0859	0.1093	0.0093	0.0912 - 0.1275
8 1970	0.0794	0.0048	0.0700 - 0.0888	0.1047	0.0102	0.0848 - 0.1246
9 1971	0.0534	0.0043	0.0450 - 0.0618	0.1076	0.0165	0.0753 - 0.1400
	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
	$\bar{F}$ = 0.0646	0.0026	0.0595 - 0.0696	$\bar{F}'$ = 0.1048	0.0035	0.0979 - 0.1118

S SURVIVAL RATE FOR ADULTS			S' SURVIVAL RATE FOR YOUNG		
ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
0.6515	0.0120	0.6280 - 0.6751	0.5415	0.0251	0.4922 - 0.5907

Example 3.5d

SELECTED ESTIMATED COVARIANCES AND CORRELATIONS UNDER H02

I	YEAR	COV(F(I),S)	CORR(F(I),S)	COV(F(I),S')	CORR(F(I),S')
1	1963	-0.00000251	-0.0157	0.00000250	0.0075
2	1964	-0.00000545	-0.0591	-0.00003531	-0.1833
3	1965	-0.00000770	-0.1242	-0.00002093	-0.1615
4	1966	-0.00001287	-0.2085	-0.00003055	-0.2367
5	1967	-0.00001325	-0.2468	-0.00002809	-0.2503
6	1968	-0.00001622	-0.3039	-0.00002999	-0.2686
7	1969	-0.00001984	-0.3609	-0.00003200	-0.2784
8	1970	-0.00002403	-0.4176	-0.00003428	-0.2849
9	1971	-0.00002176	-0.4212	-0.00002673	-0.2474

I	YEAR	COV(F'(I),S)	CORR(F'(I),S)	COV(F'(I),S')	CORR(F'(I),S')
1	1963	-0.00000199	-0.0185	-0.00000736	-0.0327
2	1964	-0.00000250	-0.0157	-0.00001118	-0.0336
3	1965	-0.00000071	-0.0077	-0.00000549	-0.0283
4	1966	-0.00000048	-0.0042	-0.00000922	-0.0384
5	1967	0.00000040	0.0041	-0.00000643	-0.0313
6	1968	0.00000126	0.0120	-0.00000655	-0.0297
7	1969	0.00000184	0.0166	-0.00000515	-0.0222
8	1970	0.00000123	0.0101	-0.00000207	-0.0081
9	1971	0.0	0.0	0.0	0.0

I	YEAR	COV(F(I),F'(I))	CORR(F(I),F'(I))	COV(F(I+1),F'(I))	CORR(F(I+1),F'(I))
1	1963	0.00000012	0.0010	-0.00000239	-0.0347
2	1964	0.00000087	0.0085	-0.00000168	-0.0246
3	1965	0.00000026	0.0066	-0.00000070	-0.0177
4	1966	0.00000054	0.0110	-0.00000085	-0.0199
5	1967	0.00000028	0.0077	-0.00000060	-0.0165
6	1968	0.00000021	0.0053	-0.00000080	-0.0199
7	1969	0.00000005	0.0012	-0.00000110	-0.0248
8	1970	-0.00000007	-0.0014	-0.00000093	-0.0212
9	1971	0.0	0.0	0.0	0.0

I	YEAR	COV(F(I),F'(I+1))	CORR(F(I),F'(I+1))	COV(F'(I),F'(I+1))	CORR(F'(I),F'(I+1))
1	1963	0.00000006	0.0004	0.00000027	0.0023
2	1964	0.00000037	0.0063	0.00000019	0.0019
3	1965	0.00000037	0.0074	0.00000014	0.0019
4	1966	0.00000030	0.0070	0.00000016	0.0020
5	1967	0.00000020	0.0050	0.00000012	0.0017
6	1968	0.00000006	0.0015	0.00000012	0.0015
7	1969	-0.00000005	-0.0010	0.00000007	0.0007
8	1970	0.0	0.0	0.0	0.0

I	YEAR	COV(F(I),F(I+1))	CORR(F(I),F(I+1))	COV(F(I),F(I+2))	CORR(F(I),F(I+2))
1	1963	-0.00000183	-0.0179	-0.00000043	-0.0063
2	1964	0.00000031	0.0078	0.00000150	0.0380
3	1965	0.00000098	0.0368	0.00000129	0.0559
4	1966	0.00000201	0.0876	0.00000256	0.1120
5	1967	0.00000235	0.1184	0.00000292	0.1428
6	1968	0.00000323	0.1584	0.00000395	0.1856
7	1969	0.00000442	0.2013	0.00000399	0.2026
8	1970	0.00000458	0.2219		

COV(S,S')	CORR(S,S')
0.00000629	0.0209

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

NUMBER OF ITERATIONS COMPLETED = 4

Example 3.5e

```
                                                     HC2
MATRIX OF DATA VALUES -- ADULTS
------------------------------------------------------------
     1     10.00  13.00   6.00   1.00   1.00   3.00   1.00   2.00   0.0
     2      0.0   58.00  21.00  16.00  15.00  13.00   6.00   1.00   1.00
     3      0.0    0.0   54.00  39.00  23.00  18.00  11.00  10.00   6.00
     4      0.0    0.0    0.0   44.00  21.00  22.00   9.00   9.00   3.00
     5      0.0    0.0    0.0    0.0   55.00  39.00  23.00  11.00  12.00
     6      0.0    0.0    0.0    0.0    0.0   66.00  46.00  29.00  18.00
     7      0.0    0.0    0.0    0.0    0.0    0.0  101.00  59.00  30.00
     8      0.0    0.0    0.0    0.0    0.0    0.0    0.0   97.00  22.00
     9      0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0   21.00


MATRIX OF EXPECTED VALUES -- ADULTS
------------------------------------------------------------
     1      9.91  12.11   5.57   4.12   2.50   1.81   1.36   0.91   0.40
     2      0.0   52.21  24.04  17.75  10.76   7.82   5.86   3.94   1.73
     3      0.0    0.0   50.31  37.14  22.52  16.37  12.27   8.25   3.61
     4      0.0    0.0    0.0   38.00  23.05  16.75  12.56   8.44   3.70
     5      0.0    0.0    0.0    0.0   56.54  41.08  30.80  20.70   9.07
     6      0.0    0.0    0.0    0.0    0.0   72.01  53.99  36.29  15.90
     7      0.0    0.0    0.0    0.0    0.0    0.0   96.19  64.65  28.33
     8      0.0    0.0    0.0    0.0    0.0    0.0    0.0   74.46  32.63
     9      0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0   16.66


MATRIX OF STANDARD NORMAL DEVIATES -- ADULTS
------------------------------------------------------------
     1      0.03   0.26   0.18  -1.55  -0.95   0.88  -0.31   1.14  -0.63
     2      0.0    0.84  -0.63  -0.42   1.30   1.86   0.06  -1.49  -0.55
     3      0.0    0.0    0.54   0.31   0.10   0.41  -0.37   0.61   1.26
     4      0.0    0.0    0.0    1.01  -0.43   1.30  -1.01   0.19  -0.36
     5      0.0    0.0    0.0    0.0   -0.21  -0.33  -1.43  -2.16   0.98
     6      0.0    0.0    0.0    0.0    0.0   -0.73  -1.12  -1.23   0.53
     7      0.0    0.0    0.0    0.0    0.0    0.0    0.51  -0.72   0.32
     8      0.0    0.0    0.0    0.0    0.0    0.0    0.0    2.72  -1.89
     9      0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    1.09


                                                     H02
MATRIX OF DATA VALUES -- YOUNG
------------------------------------------------------------
     1     83.00  35.00  18.00  16.00   6.00   8.00   5.00   3.00   1.00
     2      0.0  103.00  21.00  13.00  11.00   8.00   6.00   6.00   0.0
     3      0.0    0.0   82.00  36.00  26.00  24.00  15.00  18.00   4.00
     4      0.0    0.0    0.0  153.00  39.00  22.00  21.00  16.00   8.00
     5      0.0    0.0    0.0    0.0  109.00  38.00  31.00  15.00   1.00
     6      0.0    0.0    0.0    0.0    0.0  113.00  64.00  29.00  22.00
     7      0.0    0.0    0.0    0.0    0.0    0.0  124.00  45.00  22.00
     8      0.0    0.0    0.0    0.0    0.0    0.0    0.0   95.00  25.00
     9      0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0   38.00


MATRIX OF EXPECTED VALUES -- YOUNG
------------------------------------------------------------
     1     82.27  41.90  19.29  14.24   8.64   6.28   4.71   3.16   1.39
     2      0.0  103.06  21.61  15.95   9.67   7.03   5.27   3.54   1.55
     3      0.0    0.0   83.34  39.48  23.94  17.40  13.04   8.77   3.84
     4      0.0    0.0    0.0  152.57  38.99  28.33  21.24  14.28   6.26
     5      0.0    0.0    0.0    0.0  106.81  43.41  32.55  21.88   9.59
     6      0.0    0.0    0.0    0.0    0.0  115.21  48.12  32.35  14.17
     7      0.0    0.0    0.0    0.0    0.0    0.0  123.66  48.61  21.30
     8      0.0    0.0    0.0    0.0    0.0    0.0    0.0   94.87  26.19
     9      0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0   38.00


MATRIX OF STANDARD NORMAL DEVIATES -- YOUNG
------------------------------------------------------------
     1      0.08  -1.09  -0.30   0.47  -0.90   0.69   0.14  -0.09  -0.33
     2      0.0   -0.01  -0.13  -0.75   0.43   0.37   0.32   1.31  -1.25
     3      0.0    0.0   -0.15  -0.56   0.43   1.60   0.54   3.13   0.08
     4      0.0    0.0    0.0    0.04   0.00  -1.20  -0.05   0.46   0.70
     5      0.0    0.0    0.0    0.0    0.22  -0.84  -0.27  -1.48  -2.78
     6      0.0    0.0    0.0    0.0    0.0   -0.22   2.34  -0.60   2.09
     7      0.0    0.0    0.0    0.0    0.0    0.0    0.03  -0.53   0.15
     8      0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.01  -0.24
     9      0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.00


THESE STANDARD NORMAL DEVIATES ARE USEFUL IN EXAMINING THE AGREEMENT BETWEEN THE MODEL AND THE OBSERVED DATA IN A PARTICULAR
CELL. FOR EXAMPLE, A VALUE OF SAY -5 FOR A PARTICULAR CELL MAY INDICATE AN UNUSUAL OBSERVATION OR, PERHAPS, A MISTAKE WAS MADE
IN SUMMARIZING THE DATA. IF THE MODEL IS CORRECT, ABOUT 95% OF THESE STATISTICS SHOULD LIE WITHIN THE INTERVAL -2 TO 2.


CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H02.
-------------------------------------------------------

CHI-SQUARE VALUE =   83.57
DEGREES OF FREEDOM =       64
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN     83.57  = 0.05082


LIKELIHOOD RATIO TEST OF H01 VS H02.
------------------------------------

THIS TEST COMPARES THE MODEL UNDER H01 WITH THAT UNDER H02 AND THUS TESTS THE ASSUMPTION THAT ADULT
AND YOUNG RECOVERY RATES ARE CONSTANT FROM YEAR TO YEAR.  A 'LARGE' CHI-SQUARE VALUE INDICATES THAT
H02 BETTER DESCRIBES THE DATA AND THAT RECOVERY RATES ARE NOT CONSTANT FROM YEAR TO YEAR.

CHI-SQUARE VALUE =   77.35
DEGREES OF FREEDOM =       16
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN     77.35  = 0.0
```

Example 3.5f

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

THE HYPOTHESIS H1. (SEE BROWNIE AND ROBSON, 1974. CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

ASSUMPTIONS: (1) ANNUAL SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC.

(2) YOUNG BIRDS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES FROM THOSE OF ADULTS.

PARAMETERS:

F(I) = BAND RECOVERY RATE FOR ADULTS IN YEAR I.
S(I) = SURVIVAL RATE FOR ADULTS IN YEAR I.
F'(I) = BAND RECOVERY RATE FOR YOUNG IN YEAR I.
S'(I) = SURVIVAL RATE FOR YOUNG IN YEAR I.

STRUCTURE OF THE MODEL UNDER H1 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS

N(1)F(1)	N(1)S(1)F(2)	N(1)S(1)S(2)F(3)	N(1)S(1)S(2)S(3)F(4)
	N(2)F(2)	N(2)S(2)F(3)	N(2)S(2)S(3)F(4)
		N(3)F(3)	N(3)S(3)F(4)

BANDED AS YOUNG

M(1)F'(1)	M(1)S'(1)F(2)	M(1)S'(1)S(2)F(3)	M(1)S'(1)S(2)S(3)F(4)
	M(2)F'(2)	M(2)S'(2)F(3)	M(2)S'(2)S(3)F(4
		M(3)F'(3)	M(3)S'(3)F(4)

ESTIMATES UNDER H1

I	YR	F(I) ESTIMATE	F(I) STANDARD ERROR	F(I) 95% CONFIDENCE INTERVAL	S(I) ESTIMATE	S(I) STANDARD ERROR	S(I) 95% CONFIDENCE INTERVAL
1	1963	0.0433	0.0134	0.0170 - 0.0695	0.5756	0.1134	0.3533 - 0.7978
2	1964	0.0856	0.0092	0.0676 - 0.1036	0.6359	0.0756	0.4878 - 0.7840
3	1965	0.0590	0.0061	0.0470 - 0.0710	0.6665	0.0787	0.5122 - 0.8207
4	1966	0.0628	0.0067	0.0496 - 0.0760	0.8051	0.0977	0.6136 - 0.9967
5	1967	0.0520	0.0050	0.0422 - 0.0619	0.6496	0.0724	0.5078 - 0.7914
6	1968	0.0633	0.0055	0.0525 - 0.0740	0.5525	0.0581	0.4387 - 0.6664
7	1969	0.0789	0.0061	0.0670 - 0.0908	0.5719	0.0663	0.4419 - 0.7020
8	1970	0.0888	0.0080	0.0730 - 0.1046	0.5415	0.1286	0.2894 - 0.7936
9	1971	0.0673	0.0142	0.0395 - 0.0951			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$		= 0.0668	0.0029	0.0610 - 0.0726	$\bar{S}$ = 0.6248	0.0214	0.5828 - 0.6668

I	YR	F'(I) ESTIMATE	F'(I) STANDARD ERROR	F'(I) 95% CONFIDENCE INTERVAL	S'(I) ESTIMATE	S'(I) STANDARD ERROR	S'(I) 95% CONFIDENCE INTERVAL
1	1963	0.0863	0.0091	0.0685 - 0.1040	0.4709	0.0594	0.3545 - 0.5874
2	1964	0.1467	0.0134	0.1205 - 0.1729	0.5064	0.0699	0.3694 - 0.6434
3	1965	0.0724	0.0077	0.0573 - 0.0875	0.5891	0.0717	0.4486 - 0.7297
4	1966	0.1274	0.0096	0.1085 - 0.1463	0.5909	0.0716	0.4506 - 0.7312
5	1967	0.0909	0.0083	0.0746 - 0.1072	0.4776	0.0610	0.3581 - 0.5971
6	1968	0.0978	0.0087	0.0807 - 0.1150	0.6521	0.0723	0.5104 - 0.7939
7	1969	0.1096	0.0093	0.0914 - 0.1278	0.4635	0.0678	0.3307 - 0.5964
8	1970	0.1049	0.0102	0.0849 - 0.1248	0.3926	0.1133	0.1705 - 0.6147
9	1971	0.1076	0.0165	0.0753 - 0.1400			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$		= 0.1049	0.0035	0.0979 - 0.1118	$\bar{S}'$ = 0.5179	0.0265	0.4659 - 0.5699

Example 3.5g

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO 33570

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H1

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1963	-0.000107861	-0.071052198	-0.002229831	-0.260315312
2	1964	0.000113888	0.164048642	-0.002153369	-0.362159372
3	1965	0.000093530	0.194625663	-0.004058998	-0.527787699
4	1966	0.000277319	0.420584456	-0.003181363	-0.449952175
5	1967	0.000145477	0.398951049	-0.001924280	-0.457790708
6	1968	0.000130008	0.407034557	-0.001410437	-0.365958111
7	1969	0.000132313	0.329578633	-0.002272297	-0.266319993
8	1970	0.000256653	0.248228325		

I	YR	COVAR(F(I+1),S(I))	CORR(F(I+1),S(I))	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))
1	1963	-0.000300121	-0.288098030	-0.000042236	-0.078513900
2	1964	-0.000190633	-0.413183861	-0.000105842	-0.113419513
3	1965	-0.000316565	-0.596183595	-0.000037700	-0.068248954
4	1966	-0.000254852	-0.517437047	-0.000062679	-0.090986152
5	1967	-0.000220346	-0.553958592	-0.000036215	-0.071550000
6	1968	-0.000194598	-0.553483970	-0.000055240	-0.087394643
7	1969	-0.000372666	-0.698801335	-0.000044936	-0.071363550
8	1970	-0.001618715	-0.887260327	-0.000045436	-0.039394739

I	YR	COVAR(S'(I),S(I))	CORR(S'(I),S(I))	COVAR(S'(I),S(I+1))	CORR(S'(I),S(I+1))
1	1963	0.001651412	0.245122223	-0.001824454	-0.406359058
2	1964	0.001636182	0.309903856	-0.001714925	-0.311848868
3	1965	0.002970106	0.526383420	-0.003588038	-0.512085954
4	1966	0.002893728	0.413544294	-0.002334868	-0.450695864
5	1967	0.001663408	0.377116138	-0.001414800	-0.399447721
6	1968	0.001608233	0.382845799	-0.001664660	-0.347049369
7	1969	0.001945246	0.432637488	-0.001841692	-0.211269655
8	1970	0.009441458	0.647830523		

I	YR	COVAR(S'(I),F(I+1))	CORR(S'(I),F(I+1))
1	1963	-0.000245560	-0.449728519
2	1964	-0.000151819	-0.355785182
3	1965	-0.000279834	-0.578446973
4	1966	-0.000187041	-0.518292501
5	1967	-0.000162006	-0.483359803
6	1968	-0.000229674	-0.524885804
7	1969	-0.000302045	-0.554353714
8	1970	-0.001173591	-0.730147293

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

COVAR($\bar{S}$,$\bar{F}$) = -0.000033711 CORR($\bar{S}$,$\bar{F}$) = -0.533301536

COVAR($\bar{S}$',$\bar{F}$')= -0.000005976 CORR($\bar{S}$',$\bar{F}$')= -0.063481008

Example 3.5h

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

MATRIX OF DATA VALUES -- ADULTS H1

1	10.00	13.00	6.00	1.00	1.00	3.00	1.00	2.00	0.0
2	0.0	58.00	21.00	16.00	15.00	13.00	6.00	1.00	1.00
3	0.0	0.0	54.00	39.00	23.00	18.00	11.00	10.00	6.00
4	0.0	0.0	0.0	44.00	21.00	22.00	9.00	9.00	3.00
5	0.0	0.0	0.0	0.0	55.00	39.00	23.00	11.00	12.00
6	0.0	0.0	0.0	0.0	0.0	66.00	46.00	29.00	18.00
7	0.0	0.0	0.0	0.0	0.0	0.0	101.00	59.00	30.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	97.00	22.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.00

MATRIX OF EXPECTED VALUES -- ADULTS

1	10.00	11.45	5.04	3.60	2.42	1.92	1.33	0.86	0.37
2	0.0	55.54	24.47	17.49	11.74	9.32	6.45	4.18	1.79
3	0.0	0.0	52.22	37.32	25.05	19.89	13.77	8.93	3.83
4	0.0	0.0	0.0	37.05	24.87	19.75	13.67	8.86	3.80
5	0.0	0.0	0.0	0.0	49.07	38.97	26.97	17.49	7.50
6	0.0	0.0	0.0	0.0	0.0	68.14	47.17	30.58	13.11
7	0.0	0.0	0.0	0.0	0.0	0.0	98.64	63.95	27.41
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	83.30	35.70
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.00

MATRIX OF STANDARD NORMAL DEVIATES -- ADULTS

1	0.00	0.47	0.43	-1.38	-0.92	0.78	-0.29	1.23	-0.61
2	0.0	0.34	-0.72	-0.36	0.96	1.21	-0.18	-1.56	-0.59
3	0.0	0.0	0.25	0.28	-0.42	-0.43	-0.75	0.36	1.11
4	0.0	0.0	0.0	1.18	-0.79	0.52	-1.28	0.05	-0.41
5	0.0	0.0	0.0	0.0	0.87	0.01	-0.78	-1.57	1.65
6	0.0	0.0	0.0	0.0	0.0	-0.27	-0.17	-0.29	1.36
7	0.0	0.0	0.0	0.0	0.0	0.0	0.25	-0.64	0.50
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.57	-2.34
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00

MATRIX OF DATA VALUES -- YOUNG H1

1	83.00	35.00	18.00	16.00	6.00	8.00	5.00	3.00	1.00
2	0.0	103.00	21.00	13.00	11.00	8.00	6.00	6.00	0.0
3	0.0	0.0	82.00	36.00	26.00	24.00	15.00	18.00	4.00
4	0.0	0.0	0.0	153.00	39.00	22.00	21.00	16.00	8.00
5	0.0	0.0	0.0	0.0	109.00	38.00	31.00	15.00	1.00
6	0.0	0.0	0.0	0.0	0.0	113.00	64.00	29.00	22.00
7	0.0	0.0	0.0	0.0	0.0	0.0	124.00	45.00	22.00
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	95.00	25.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.00

MATRIX OF EXPECTED VALUES -- YOUNG

1	83.00	39.01	17.19	12.28	8.25	6.55	4.53	2.94	1.26
2	0.0	103.00	21.08	15.07	10.11	8.03	5.56	3.60	1.54
3	0.0	0.0	82.00	42.19	28.33	22.49	15.57	10.09	4.33
4	0.0	0.0	0.0	153.00	37.16	29.50	20.42	13.24	5.67
5	0.0	0.0	0.0	0.0	109.00	36.43	25.22	16.35	7.01
6	0.0	0.0	0.0	0.0	0.0	113.00	59.70	38.71	16.59
7	0.0	0.0	0.0	0.0	0.0	0.0	124.00	46.90	20.10
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	95.00	25.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.00

MATRIX OF STANDARD NORMAL DEVIATES -- YOUNG

1	0.0	-0.66	0.20	1.07	-0.79	0.57	0.22	0.04	-0.23
2	0.0	0.0	-0.02	-0.54	0.28	-0.01	0.19	1.27	-1.24
3	0.0	0.0	0.0	-0.97	-0.44	0.32	-0.15	2.50	-0.16
4	0.0	0.0	0.0	0.0	0.31	-1.40	0.13	0.76	0.98
5	0.0	0.0	0.0	0.0	0.0	0.26	1.16	-0.34	-2.28
6	0.0	0.0	0.0	0.0	0.0	0.0	0.57	-1.59	1.34
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.28	0.43
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Example 3.5i

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

THE HYPOTHESIS H2. (SEE BROWNIE AND ROBSON, 1974. CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

ASSUMPTIONS: (1) ANNUAL SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC.

(2) YOUNG BIRDS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES FROM THOSE OF ADULTS.

(3) IN ANY YEAR, THE REPORTING RATE FOR NEW RELEASES IS DIFFERENT FROM THAT FOR SURVIVORS OF PREVIOUSLY BANDED COHORTS, AND HENCE THE CORRESPONDING RECOVERY RATES ARE DIFFERENT.

H2 IS AN EXTENSION OF H1 IN THAT THE FIRST YEAR ADULT RECOVERY RATE IN YEAR I IS DIFFERENT FROM THE RECOVERY RATE IN YEAR I OF PREVIOUSLY BANDED ADULTS. (THE SOLICITING OF BANDS FROM HUNTERS BY CONSERVATION OFFICERS NEAR BANDING SITES MAY GIVE RISE TO THIS SITUATION).

PARAMETERS:

F'''(I)= BAND RECOVERY RATE IN YEAR I FOR ADULTS BANDED IN YEAR I.
F(I) = BAND RECOVERY RATE IN YEAR I FOR SURVIVORS OF COHORTS BANDED BEFORE YEAR I.
S(I) = SURVIVAL RATE FOR ADULTS IN YEAR I.
F'(I) = BAND RECOVERY RATE FOR YOUNG IN YEAR I.
S'(I) = SURVIVAL RATE FOR YOUNG IN YEAR I.

STRUCTURE OF THE MODEL UNDER H2 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS

N(1)F'''(1)	N(1)S(1)F(2)	N(1)S(1)S(2)F(3)	N(1)S(1)S(2)S(3)F(4)
	N(2)F'''(2)	N(2)S(2)F(3)	N(2)S(2)S(3)F(4)
		N(3)F'''(3)	N(3)S(3)F(4)

BANDED AS YOUNG

M(1)F'(1)	M(1)S'(1)F(2)	M(1)S'(1)S(2)F(3)	M(1)S'(1)S(2)S(3)F(4)
	M(2)F'(2)	M(2)S'(2)F(3)	M(2)S'(2)S(3)F(4
		M(3)F'(3)	M(3)S'(3)F(4)

ESTIMATES UNDER H2

I	YR	F(I) ESTIMATE	F(I) STANDARD ERROR	F(I) 95% CONFIDENCE INTERVAL	S(I) ESTIMATE	S(I) STANDARD ERROR	S(I) 95% CONFIDENCE INTERVAL
1	1963				0.6126	0.1377	0.3426 - 0.8825
2	1964	0.0760	0.0165	0.0437 - 0.1084	0.6314	0.0949	0.4454 - 0.8173
3	1965	0.0558	0.0097	0.0367 - 0.0749	0.7427	0.1137	0.5199 - 0.9655
4	1966	0.0521	0.0084	0.0356 - 0.0686	0.7900	0.1270	0.5411 - 1.0389
5	1967	0.0457	0.0067	0.0326 - 0.0588	0.5858	0.0873	0.4146 - 0.7569
6	1968	0.0660	0.0091	0.0483 - 0.0838	0.5833	0.0876	0.4116 - 0.7549
7	1969	0.0753	0.0104	0.0549 - 0.0957	0.9766	0.2392	0.5077 - 1.4455
8	1970	0.0464	0.0110	0.0248 - 0.0679			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
		$\bar{F}$ = 0.0596	0.0040	0.0517 - 0.0675	$\bar{S}$ = 0.7032	0.0371	0.6304 - 0.7760

Example 3.5j

F'(I)

I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	
1	1963	0.0863	0.0091	0.0685 -	0.1040
2	1964	0.1467	0.0134	0.1205 -	0.1729
3	1965	0.0724	0.0077	0.0573 -	0.0875
4	1966	0.1274	0.0096	0.1085 -	0.1463
5	1967	0.0909	0.0083	0.0746 -	0.1072
6	1968	0.0978	0.0087	0.0807 -	0.1150
7	1969	0.1096	0.0093	0.0914 -	0.1278
8	1970	0.1049	0.0102	0.0849 -	0.1248
9	1971	0.1076	0.0165	0.0753 -	0.1400
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	
$\bar{F}'$		= 0.1049	0.0035	0.0979 -	0.1118

S'(I)

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	
0.5012	0.0834	0.3378 -	0.6646
0.5197	0.0812	0.3607 -	0.6788
0.6675	0.1000	0.4714 -	0.8635
0.6428	0.0921	0.4624 -	0.8232
0.4607	0.0690	0.3254 -	0.5960
0.6726	0.0964	0.4836 -	0.8615
0.8125	0.2049	0.4108 -	1.2142
AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	
$\bar{S}'$ = 0.6110	0.0424	0.5279 -	0.6941

F'''(I)

I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	
1	1963	0.0433	0.0134	0.0170 -	0.0695
2	1964	0.0894	0.0112	0.0674 -	0.1113
3	1965	0.0610	0.0080	0.0452 -	0.0768
4	1966	0.0746	0.0108	0.0534 -	0.0958
5	1967	0.0583	0.0076	0.0434 -	0.0733
6	1968	0.0613	0.0073	0.0470 -	0.0756
7	1969	0.0808	0.0077	0.0657 -	0.0959
8	1970	0.1034	0.0099	0.0839 -	0.1229
9	1971	0.0673	0.0142	0.0395 -	0.0951
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	
$\bar{F}'''$		= 0.0710	0.0034	0.0643 -	0.0778

SK-1 FK

ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	
0.0235	0.0086	0.0067 -	0.0402

Example 3.5k

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H2

I	YR	COVAR(F'''(I),F(I))	CORR(F'''(I),F(I))	COVAR(F'''(I),S(I))	CORR(F'''(I),S(I))
1	1963			-0.000114794	-0.062251556
2	1964	-0.000010471	-0.056676081	-0.000086939	-0.081830862
3	1965	-0.000003847	-0.049178047	-0.000051205	-0.055976879
4	1966	-0.000006584	-0.072254022	-0.000099861	-0.072709400
5	1967	-0.000002827	-0.055506824	-0.000036230	-0.054358936
6	1968	-0.000003757	-0.056670512	-0.000033189	-0.051852051
7	1969	-0.000004868	-0.060706176	-0.000063125	-0.034231498
8	1970	-0.000005111	-0.046794813		

I	YR	COVAR(F'''(I+1),S(I))	CORR(F'''(I+1),S(I))	COVAR(F'''(I+1),S'(I))	CORR(F'''(I+1),S'(I))
1	1963	0.000084350	0.054695350	0.000069015	0.073926676
2	1964	0.000043530	0.057022187	0.000035833	0.054874386
3	1965	0.000093876	0.076346333	0.000084367	0.077975828
4	1966	0.000048864	0.050420309	0.000039758	0.056593216
5	1967	0.000033330	0.052220432	0.000026214	0.051956476
6	1968	0.000037704	0.055849972	0.000043474	0.058497243
7	1969	0.000107663	0.045265442	0.000089578	0.043964100

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(S(I),F(I+1))	CORR(S(I),F(I+1))
1	1963			-0.000566317	-0.249235131
2	1964	0.000583705	0.372885923	-0.000289453	-0.313774667
3	1965	0.000340492	0.308022918	-0.000538855	-0.562594553
4	1966	0.000573209	0.535794469	-0.000386581	-0.456110834
5	1967	0.000286628	0.491740167	-0.000379999	-0.479641717
6	1968	0.000378391	0.476257935	-0.000458447	-0.503149047
7	1969	0.000767553	0.308389525	-0.002009567	-0.764648386

I	YR	COVAR(F(I+1),S'(I))	CORR(F(I+1),S'(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1963	-0.000463363	-0.336868308	-0.004701939	-0.359854127
2	1964	-0.000238274	-0.301956213	-0.003852440	-0.357154159
3	1965	-0.000484274	-0.574602458	-0.008173466	-0.566140566
4	1966	-0.000314540	-0.511951969	-0.004953686	-0.446678382
5	1967	-0.000298865	-0.477217070	-0.003356640	-0.438859686
6	1968	-0.000528613	-0.526998331	-0.005944528	-0.283719809
7	1969	-0.001671995	-0.742665819		

I	YR	COVAR(S(I),S'(I))	CORR(S(I),S'(I))	COVAR(S(I+1),S'(I))	CORR(S(I+1),S'(I))
1	1963	0.005476698	0.477006912	-0.003847140	-0.486381933
2	1964	0.003420540	0.444227717	-0.003171275	-0.343701632
3	1965	0.007543445	0.663253254	-0.007345557	-0.578223844
4	1966	0.006046373	0.517247460	-0.004030544	-0.501364979
5	1967	0.003109774	0.515808581	-0.002639961	-0.436641458
6	1968	0.004990235	0.590980631	-0.006854348	-0.297168051
7	1969	0.039337896	0.802354321		

I	YR	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))
1	1963	-0.000044950	-0.059560774
2	1964	-0.000108627	-0.100226310
3	1965	-0.000042712	-0.055417230
4	1966	-0.000068185	-0.076990774
5	1967	-0.000034931	-0.060945499
6	1968	-0.000056970	-0.067593315
7	1969	-0.000078765	-0.041369166

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

Example 3.51

MALE MALLARDS BANDEC PRESEASON IN THE SAN LUIS VALLEY, COLORADO

MATRIX OF DATA VALUES -- ADULTS H2

1	10.00	13.00	6.00	1.00	1.00	3.00	1.00	2.0C	0.0
2	0.0	58.00	21.00	16.00	15.00	13.00	6.00	1.00	1.00
3	0.0	C.0	54.00	39.00	23.00	18.00	11.00	10.00	6.00
4	0.0	0.0	0.0	44.00	21.00	22.00	9.00	9.00	3.00
5	0.0	C.0	0.0	0.0	55.0C	39.00	23.00	11.00	12.0C
6	0.0	0.0	0.0	0.0	0.0	66.00	46.00	29.00	18.00
7	0.0	C.0	0.0	0.0	C.0	0.0	101.00	59.00	30.0C
8	0.0	C.0	0.0	0.0	C.0	0.0	0.0	97.00	22.00
9	0.0	0.0	0.0	0.0	C.0	0.0	0.0	0.0	21.00

MATRIX OF EXPECTED VALUES -- ADULTS

1	10.00	1C.89	5.09	3.58	2.51	2.14	1.44	0.9C	0.46
2	0.0	58.00	23.05	16.20	11.35	9.70	6.52	4.09	2.07
3	0.0	C.0	54.00	34.71	24.32	20.78	13.97	8.77	4.44
4	0.0	0.0	0.0	44.00	21.54	18.40	12.37	7.77	3.93
5	0.0	0.0	0.0	0.0	55.0C	36.83	24.76	15.54	7.86
6	0.0	0.0	0.0	0.0	0.0	66.00	47.81	30.01	15.18
7	0.0	0.0	0.0	0.0	0.0	0.0	101.00	59.10	29.9C
8	0.0	C.0	0.0	0.0	C.0	0.0	0.0	97.00	22.0C
9	0.0	0.0	0.0	0.0	C.0	0.0	0.0	0.0	21.00

MATRIX OF STANCARD NORMAL DEVIATES -- ADULTS

1	0.00	C.65	0.41	-1.37	-C.96	0.59	-0.37	1.16	-0.68
2	0.0	C.00	-0.44	-0.05	1.09	1.07	-0.21	-1.53	-0.75
3	0.0	0.0	0.00	0.74	-0.27	-0.62	-0.80	0.42	0.74
4	0.0	0.0	0.0	0.00	-C.12	0.85	-0.97	0.45	-0.47
5	0.0	C.0	0.0	0.0	0.00	0.36	-0.36	-1.16	1.48
6	0.0	C.0	0.0	0.0	C.0	0.00	-0.27	-0.19	0.73
7	0.0	C.0	0.0	0.0	C.0	0.0	0.00	-0.01	0.02
8	0.0	0.0	C.0	0.0	0.0	0.0	0.0	0.00	0.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0C

MATRIX OF DATA VALUES -- YOUNG H2

1	83.00	35.00	18.00	16.00	6.00	8.00	5.00	3.00	1.0C
2	0.0	103.00	21.00	13.00	11.00	8.00	6.00	6.00	0.0
3	0.0	C.0	82.00	36.00	26.00	24.00	15.00	18.00	4.00
4	0.0	0.0	0.0	153.00	39.0C	22.00	21.00	16.00	8.00
5	0.0	C.0	0.0	0.0	109.00	38.00	31.00	15.0C	1.00
6	0.0	0.0	0.0	0.0	0.0	113.00	64.00	29.00	22.00
7	0.0	0.0	0.0	0.0	C.0	0.0	124.00	45.00	22.00
8	0.0	0.0	0.0	0.0	C.0	0.0	0.0	95.00	25.00
9	0.0	C.0	0.0	0.0	0.0	0.0	0.0	0.0	38.00

MATRIX OF EXPECTED VALUES -- YOUNG

1	83.0C	37.11	17.33	12.18	8.54	7.30	4.90	3.08	1.56
2	0.0	103.00	20.53	14.43	10.11	8.64	5.81	3.65	1.84
3	0.0	0.0	82.00	39.90	27.96	23.89	16.06	10.08	5.10
4	0.0	0.0	0.0	153.00	35.67	30.48	20.49	12.86	6.51
5	0.0	0.0	0.0	0.0	109.00	36.83	24.76	15.54	7.86
6	0.0	0.0	0.0	0.0	0.0	113.00	59.11	37.11	18.78
7	0.0	0.0	0.0	0.0	C.0	0.0	124.00	44.49	22.51
8	0.0	C.0	0.0	0.0	0.0	0.0	0.0	95.00	25.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	38.00

MATRIX OF STANCARD NORMAL DEVIATES -- YOUNG

1	0.0	-0.35	0.16	1.10	-0.87	0.26	0.04	-0.04	-0.45
2	0.0	C.0	0.11	-0.38	0.28	-0.22	0.08	1.24	-1.36
3	0.0	C.0	0.0	-0.63	-0.38	0.02	-0.27	2.5C	-0.49
4	0.0	0.0	0.0	0.0	0.57	-1.56	0.11	0.88	0.59
5	0.0	C.0	0.0	0.0	C.0	0.20	1.27	-0.14	-2.46
6	0.0	0.0	0.0	0.0	0.0	0.0	0.65	-1.35	0.75
7	0.0	0.0	0.0	0.0	C.0	0.0	0.0	0.C8	-0.11
8	0.0	0.0	0.0	0.0	C.0	0.0	0.0	0.0	0.00
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.C	0.0

Example 3.5m

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

THE HYPOTHESIS H3. (SEE BROWNIE AND ROBSON, 1974. CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

ASSUMPTIONS: (1) ANNUAL SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC.

(2) SURVIVAL AND RECOVERY RATES ARE AGE-DEPENDENT FOR THE FIRST TWO YEARS OF LIFE. (THIS EMBRACES ASSUMPTION (3) OF H2 FOR THE TYPE OF DATA BEING ANALYSED.)

H3 ASSUMES THE PARAMETERS S AND F ARE AGE-SPECIFIC FOR THREE AGE CLASSES (NAMELY, YOUNG, SUBADULT, AND ADULT,) BUT ONLY TWO AGE CLASSES ARE RECOGNIZED DURING BANDING, BECAUSE SUBADULTS AND ADULTS ARE USUALLY INDISTINGUISHABLE. THUS THE ASSUMPTIONS OF H3 GIVE RISE TO A MODEL FOR WHICH MOST OF THE PARAMETERS OF INTEREST ARE NOT ESTIMABLE, HENCE NO ESTIMATES ARE COMPUTED HERE. HOWEVER THE ASSUMPTIONS OF H3 ARE EXAMINED BELOW BY MEANS OF A GOODNESS OF FIT TEST AND A TEST OF H2 AGAINST THE ALTERNATIVE H3

THE HYPOTHESIS HO.

ASSUMPTIONS: SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC BUT AGE-INDEPENDENT.

THE AGE-INDEPENDENCE ASSUMPTION OF HO IS USUALLY INAPPROPRIATE FOR BIRDS BANDED AS YOUNG AND ESTIMATION UNDER HO IS OMITTED THOUGH TESTS RELATED TO THIS HYPOTHESIS ARE COMPUTED BELOW. IF HO IS NOT REJECTED, THE DATA SHOULD BE POOLED AND ANALYZED USING THE MODELS THAT ASSUME PARAMETERS ARE AGE-INDEPENDENT.

Example 3.5n

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

TESTS TO DESCRIMINATE BETWEEN THE MODELS UNDER H0, H1, H2, H3. (SEE BROWNIE AND ROBSON, 1974, CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

H0, H1, H2 AND H3 REPRESENT A SERIES OF HYPOTHESES WITH PROGRESSIVELY MORE GENERAL ASSUMPTIONS ABOUT THE POPULATION PARAMETERS. TO DETERMINE WHICH ASSUMPTIONS ARE APPROPRIATE FOR A GIVEN DATA SET, A SERIES OF TESTS ARE CARRIED OUT, WHERE EACH TEST IN THE SERIES COMPARES A GIVEN HYPOTHESIS AGAINST A MORE GENERAL ALTERNATIVE. (THUS THE FIRST TEST IN THE SERIES COMPARES H0 AGAINST THE MORE GENERAL H1). IN EACH CASE, IF THE TEST RESULTS IN A 'LARGE' CHI-SQUARE VALUE, THEN THE RESTRICTIVE HYPOTHESIS SHOULD BE REJECTED IN FAVOR OF THE MORE GENERAL ONE, WHILE A 'SMALL' CHI-SQUARE VALUE SUGGESTS THERE IS NO REASON TO DISCARD THE SIMPLER HYPOTHESIS.

CHI-SQUARE TEST OF H0 VS H1

	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
	R(I.)	N(I)-R(I.)		W(I)	Z(I+1)	
	Q(I.)	M(I)-Q(I.)		Q(I,I)	Q(I.)-Q(I,I)	
I= 1	37	194		10	27	
	175	787	0.602	83	92	5.163
I= 2	131	518		106	144	
	168	534	2.747	103	65	14.371
I= 3	161	724		120	250	
	205	927	0.002	82	123	3.315
I= 4	108	482		165	316	
	259	942	2.581	153	106	42.149
I= 5	140	803		197	365	
	194	1005	0.714	109	85	26.731
I= 6	159	918		261	348	
	228	927	9.632	113	115	3.017
I= 7	190	1060		339	314	
	191	940	1.258	124	67	10.096
I= 8	119	819		350	150	
	120	786	0.127	95	25	4.014
I= 9	21	291				
	38	315	3.334			

TOTAL CHI-SQUARE WITH 17 DEGREES OF FREEDOM = 129.853

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 129.85 = 0.0

THIS TEST OF THE HYPOTHESIS H0 AGAINST THE HYPOTHESIS H1 TESTS THE ASSUMPTION THAT YOUNG AND ADULTS HAVE THE SAME SURVIVAL AND RECOVERY RATES.

Example 3.5o

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

CHI-SQUARE TEST OF H1 VS H2

	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
I= 2	58	73	
	48	71	0.396
I= 3	54	107	
	66	143	0.160
I= 4	44	64	
	121	252	2.561
I= 5	55	85	
	142	280	1.467
I= 6	66	93	
	195	255	C.160
I= 7	101	89	
	238	225	0.166
I= 8	97	22	
	253	128	9.856

TOTAL CHI-SQUARE WITH 7 DEGREES OF FREEDOM = 14.766

PROBABILITY OF A CHI-SCUARE VALUE LARGER THAN 14.77 = 0.03912

THIS TEST OF THE HYPOTHESIS H1 AGAINST THE HYPOTHESIS H2 TESTS THE ASSUMPTION THAT RECOVERY RATES FOR NEWLY RELEASED ADULTS ARE THE SAME AS FOR SURVIVORS OF PREVIOUSLY BANDED COHORTS.

CHI-SQUARE TEST OF H2 VS H3

	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
I= 1	35	57	
	13	14	0.886
I= 2	21	44	
	45	99	0.023
I= 3	36	87	
	85	165	0.842
I= 4	39	67	
	103	213	0.626
I= 5	38	47	
	157	208	0.080
I= 6	64	51	
	174	174	1.105
I= 7	45	22	
	208	106	0.021

TOTAL CHI-SQUARE WITH 7 DEGREES OF FREEDOM = 3.584

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 3.58 = 0.82622

REJECTION OF H2 IN FAVOR OF H3 WOULD INDICATE THAT SURVIVAL AND RECOVERY RATES ARE AGE-DEPENDENT FOR AT LEAST THE FIRST TWO YEARS.

THE HYPOTHESES H01 AND H02 ARE MORE RESTRICTIVE THAN H1 BUT ARE NOT COMPARABLE IN THIS SENSE WITH H0, THUS H01 AND H02 DO NOT FIT INTO THE ABOVE SERIES. IN PRACTICE THE MODELS UNDER H02 AND H1 ARE LIKELY TO BE OF MOST USE, SO A LIKELIHOOD RATIO TEST TO DISTINGUISH BETWEEN THESE MODELS IS COMPUTED BELOW. THIS TESTS THE ASSUMPTION THAT YOUNG AND ADULT SURVIVAL RATES ARE CONSTANT FROM YEAR TO YEAR.

LIKELIHOOD RATIO TEST OF H02 VS H1.

CHI-SQUARE VALUE = 26.94
DEGREES OF FREEDOM = 14
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 26.94 = 0.01962

Example 3.5p

MALE MALLARDS BANDED PRESEASON IN THE SAN LUIS VALLEY, COLORADO

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H3

	CONTINGENCY TABLES	CORRESPONDING CHI-SQUARE STATISTICS AND DEGREES OF FREEDOM
I= 2	23. 13. 16. 21. 15. 8. 16. 18. 4. 3. 1. 6.	3.74 WITH 6 D.F.
I= 3	16. 11. 18. 23. 39. 6. 6. 8. 11. 13. 8. 12. 24. 22. 33.	4.62 WITH 8 D.F.
I= 4	3. 9. 9. 22. 21. 4. 18. 15. 24. 26. 8. 22. 29. 50. 56.	3.31 WITH 8 D.F.
I= 5	12. 11. 23. 39. 8. 16. 21. 22. 15. 49. 53. 96.	9.76 WITH 6 D.F.
I= 6	18. 29. 46. 1. 15. 31. 35. 76. 97.	10.05 WITH 4 D.F.
I= 7	30. 59. 22. 29. 54.120.	2.58 WITH 2 D.F.

TOTAL CHI-SQUARE 34.07 WITH 34 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 34.07 = 0.46433

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H0

TOTAL CHI-SQUARE 182.27 WITH 65 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 182.27 = 0.0

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H1

TOTAL CHI-SQUARE 52.42 WITH 48 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 52.42 = 0.30656

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H2

TOTAL CHI-SQUARE 37.66 WITH 41 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 37.66 = 0.62009

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H3

TOTAL CHI-SQUARE 34.07 WITH 34 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 34.07 = 0.46433

FROM THE MODELS ABOVE, ONE SHOULD CHOOSE THE SIMPLEST MODEL (FEWEST PARAMETERS) THAT ADEQUATELY DESCRIBES THE DATA. ADEQUACY MAY BE JUDGED BY EXAMINING THE RESULTS OF (1) THE GOODNESS OF FIT TESTS, AND (2) THE TESTS BETWEEN SPECIFIC MODELS. FREQUENTLY, H02 OR H1 IS ADEQUATE.

Example 3.6a

YOUNG AND ADULT MALE MALLARDS BANDED PRESEASON IN SOUTHERN ONTARIO, 1965

ADULTS INPUT MATRIX

1965	604.	43.	20.	10.	12.	3.	5.	2.	3.	0.
1966	333.	0.	36.	7.	9.	6.	4.	2.	2.	0.
1967	703.	0.	0.	40.	24.	19.	7.	5.	3.	1.
1968	497.	0.	0.	0.	34.	13.	7.	4.	7.	1.
1969	536.	0.	0.	0.	0.	46.	13.	7.	9.	1.
1970	828.	0.	0.	0.	0.	0.	56.	28.	25.	6.
1971	1353.	0.	0.	0.	0.	0.	0.	79.	42.	17.
1972	1080.	0.	0.	0.	0.	0.	0.	0.	65.	34.

YOUNG INPUT MATRIX

1965	1570.	132.	48.	33.	13.	8.	9.	8.	5.	1.
1966	1462.	0.	175.	33.	8.	11.	10.	4.	7.	1.
1967	1611.	0.	0.	165.	39.	23.	12.	13.	5.	7.
1968	1733.	0.	0.	0.	193.	51.	24.	13.	12.	5.
1969	1848.	0.	0.	0.	0.	193.	43.	39.	15.	9.
1970	3456.	0.	0.	0.	0.	0.	367.	113.	56.	32.
1971	4488.	0.	0.	0.	0.	0.	0.	392.	176.	70.
1972	3584.	0.	0.	0.	0.	0.	0.	0.	342.	101.

BASIC SUBTOTALS

I	RROW(I)	RCOL(I)	CROW(I)	QCOL(I)	T(I)	U(I)	W(I)	Z(I)
1	98.00	43.00	257.00	132.00	98.00	257.00	43.00	0.0
2	66.00	56.00	249.00	223.00	121.00	374.00	104.00	55.00
3	99.00	57.00	264.00	231.00	164.00	415.00	123.00	142.00
4	66.00	79.00	298.00	253.00	173.00	482.00	139.00	192.00
5	76.00	87.00	299.00	286.00	170.00	528.00	180.00	218.00
6	115.00	92.00	568.00	465.00	198.00	810.00	190.00	219.00
7	138.00	127.00	638.00	582.00	244.00	983.00	317.00	250.00
8	99.00	156.00	443.00	618.00	216.00	844.00	432.00	272.00
					60.00	226.00		185.00

THE HYPOTHESIS H2. (SEE BROWNIE AND ROBSON, 1974. CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

ASSUMPTIONS: (1) ANNUAL SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC.

(2) YOUNG BIRDS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES FROM THOSE OF ADULTS.

(3) IN ANY YEAR, THE REPORTING RATE FOR NEW RELEASES IS DIFFERENT FROM THAT FOR SURVIVORS OF PREVIOUSLY BANDED COHORTS, AND HENCE THE CORRESPONDING RECOVERY RATES ARE DIFFERENT.

H2 IS AN EXTENSION OF H1 IN THAT THE FIRST YEAR ADULT RECOVERY RATE IN YEAR I IS DIFFERENT FROM THE RECOVERY RATE IN YEAR I OF PREVIOUSLY BANDED ADULTS. (THE SOLICITING OF BANDS FROM HUNTERS BY CONSERVATION OFFICERS NEAR BANDING SITES MAY GIVE RISE TO THIS SITUATION).

PARAMETERS:

F'''(I)= BAND RECOVERY RATE IN YEAR I FOR ADULTS BANDED IN YEAR I.
F(I) = BAND RECOVERY RATE IN YEAR I FOR SURVIVORS OF COHORTS BANDED BEFORE YEAR I.
S(I) = SURVIVAL RATE FOR ADULTS IN YEAR I.
F'(I) = BAND RECOVERY RATE FOR YOUNG IN YEAR I.
S'(I) = SURVIVAL RATE FOR YOUNG IN YEAR I.

STRUCTURE OF THE MODEL UNDER H2 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

```
                         BANDED AS ADULTS

N(1)F'''(1)  N(1)S(1)F(2)    N(1)S(1)S(2)F(3)     N(1)S(1)S(2)S(3)F(4)
             N(2)F'''(2)     N(2)S(2)F(3)         N(2)S(2)S(3)F(4)
                             N(3)F'''(3)          N(3)S(3)F(4)

                         BANDED AS YOUNG

M(1)F'(1)    M(1)S'(1)F(2)   M(1)S'(1)S(2)F(3)    M(1)S'(1)S(2)S(3)F(4)
             M(2)F'(2)       M(2)S'(2)F(3)        M(2)S'(2)S(3)F(4
                             M(3)F'(3)            M(3)S'(3)F(4)
```

Example 3.6b

ESTIMATES UNDER H2

		F(I)			S(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1965				0.6105	0.1368	0.3423 - 0.8786
2	1966	0.0547	0.0127	0.0298 - 0.0796	0.6509	0.1437	0.3692 - 0.9325
3	1967	0.0524	0.0098	0.0331 - 0.0716	0.8095	0.1749	0.4666 - 1.1524
4	1968	0.0363	0.0076	0.0214 - 0.0513	0.6526	0.1637	0.3318 - 0.9734
5	1969	0.0397	0.0083	0.0233 - 0.0560	0.4545	0.1010	0.2566 - 0.6524
6	1970	0.0500	0.0084	0.0335 - 0.0665	0.7594	0.1409	0.4832 - 1.0356
7	1971	0.0487	0.0077	0.0336 - 0.0639	0.3926	0.0873	0.2216 - 0.5636
8	1972	0.0765	0.0149	0.0473 - 0.1057			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$		= 0.0512	0.0039	0.0436 - 0.0588	$\bar{S}$ = 0.6186	0.0221	0.5753 - 0.6618

		F'(I)			S'(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1965	0.0841	0.0070	0.0703 - 0.0978	0.5338	0.1082	0.3218 - 0.7458
2	1966	0.1197	0.0085	0.1031 - 0.1363	0.3657	0.0646	0.2390 - 0.4924
3	1967	0.1024	0.0076	0.0876 - 0.1172	0.5928	0.1195	0.3585 - 0.8270
4	1968	0.1114	0.0076	0.0966 - 0.1262	0.6141	0.1268	0.3657 - 0.8626
5	1969	0.1044	0.0071	0.0905 - 0.1184	0.4658	0.0762	0.3163 - 0.6152
6	1970	0.1050	0.0052	0.0948 - 0.1151	0.6128	0.0937	0.4291 - 0.7964
7	1971	0.0873	0.0042	0.0791 - 0.0956	0.4935	0.0950	0.3074 - 0.6796
8	1972	0.0954	0.0049	0.0858 - 0.1050			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$		= 0.1012	0.0024	0.0966 - 0.1058	$\bar{S}'$ = 0.5255	0.0378	0.4515 - 0.5995

		F'''(I)			SK...SK+I-1FK+I		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1965	0.0712	0.0105	0.0507 - 0.0917	0.0315	0.0053	0.0211 - 0.0419
2	1966	0.1081	0.0170	0.0748 - 0.1415			
3	1967	0.0569	0.0087	0.0398 - 0.0740			
4	1968	0.0684	0.0113	0.0462 - 0.0906			
5	1969	0.0858	0.0121	0.0621 - 0.1095			
6	1970	0.0676	0.0087	0.0505 - 0.0847			
7	1971	0.0584	0.0064	0.0459 - 0.0709			
8	1972	0.0602	0.0072	0.0460 - 0.0744			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL			
$\bar{F}'''$		= 0.0721	0.0038	0.0647 - 0.0795			

Example 3.6c

YOUNG AND ADULT MALE MALLARDS BANDED PRESEASON IN SOUTHERN ONTARIO, 1965

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H2

I	YR	COVAR(F'''(I),F(I))	CORR(F'''(I),F(I))	COVAR(F'''(I),S(I))	CORR(F'''(I),S(I))
1	1965			-0.000071953	-0.050263757
2	1966	-0.000017757	-0.082128853	-0.000211305	-0.086413352
3	1967	-0.000004239	-0.049451699	-0.000065521	-0.042866313
4	1968	-0.000005003	-0.057858104	-0.000089832	-0.048466721
5	1969	-0.000006354	-0.062933575	-0.000072768	-0.059563022
6	1970	-0.000004083	-0.055501886	-0.000062032	-0.050443814
7	1971	-0.000002103	-0.042725413	-0.000016943	-0.030462790
8	1972	-0.000004264	-0.039589086		

I	YR	COVAR(F'''(I+1),S(I))	CORR(F'''(I+1),S(I))	COVAR(F'''(I+1),S'(I))	CORR(F'''(I+1),S'(I))
1	1965	0.000198185	0.085127837	0.000173282	0.094147318
2	1966	0.000052680	0.041959024	0.000029597	0.052408655
3	1967	0.000111429	0.056245979	0.000081591	0.060283337
4	1968	0.000104494	0.052768064	0.000098331	0.064112899
5	1969	0.000037122	0.042125834	0.000038044	0.057182092
6	1970	0.000032774	0.036485280	0.000026444	0.044278074
7	1971	0.000021879	0.034649850	0.000027501	0.040021480

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(S(I),F(I+1))	CORR(S(I),F(I+1))
1	1965			-0.001012746	-0.582562295
2	1966	0.001079793	0.591359720	-0.000529294	-0.375399742
3	1967	0.000658314	0.383517011	-0.000860296	-0.643962160
4	1968	0.000693555	0.554897300	-0.000814945	-0.596652227
5	1969	0.000567510	0.673483256	-0.000357640	-0.420106698
6	1970	0.000597624	0.503059209	-0.000599829	-0.551351409
7	1971	0.000310095	0.460341645	-0.000855720	-0.658995815

I	YR	COVAR(F(I+1),S'(I))	CORR(F(I+1),S'(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1965	-0.000885492	-0.644285909	-0.012051146	-0.612953185
2	1966	-0.000297375	-0.468890496	-0.008180991	-0.325408456
3	1967	-0.000629928	-0.690186048	-0.015446965	-0.539436014
4	1968	-0.000766877	-0.724929336	-0.009333376	-0.564697195
5	1969	-0.000366516	-0.570257835	-0.005433068	-0.381821171
6	1970	-0.000483983	-0.669112868	-0.004833199	-0.393107900
7	1971	-0.001075621	-0.761157018		

I	YR	COVAR(S(I),S'(I))	CORR(S(I),S'(I))	COVAR(S(I+1),S'(I))	CORR(S(I+1),S'(I))
1	1965	0.010981679	0.742078948	-0.010536898	-0.677896799
2	1966	0.004383184	0.471876663	-0.004596349	-0.406449248
3	1967	0.014960725	0.715472751	-0.011310611	-0.578156926
4	1968	0.013491966	0.650232249	-0.008782856	-0.686103635
5	1969	0.003788974	0.492179746	-0.005567908	-0.518288443
6	1970	0.008696396	0.658718239	-0.003899758	-0.477070797
7	1971	0.006428260	0.775924154		

I	YR	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))
1	1965	-0.000028583	-0.037732383
2	1966	-0.000029940	-0.054558622
3	1967	-0.000037685	-0.041738450
4	1968	-0.000039466	-0.041197019
5	1969	-0.000026321	-0.048530931
6	1970	-0.000018400	-0.037883746
7	1971	-0.000009604	-0.024000377

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

Example 3.6d

YOUNG AND ADULT MALE MALLARDS BANDEC PRESEASON IN SOUTHERN ONTARIO, 1965

MATRIX OF CATA VALUES -- ADULTS

1	43.00	2C.00	10.00	12.00	3.CC	5.00	2.00	3.0C	0.0
2	0.0	36.00	7.00	9.00	6.00	4.00	2.00	2.0C	0.0
3	0.0	C.0	40.00	24.00	1S.00	7.00	5.00	3.00	1.00
4	0.0	0.0	0.0	34.00	13.0C	7.00	4.00	7.00	1.00
5	0.0	0.0	0.0	0.0	46.00	13.00	7.00	9.00	1.00
6	0.0	0.0	0.0	0.0	0.0	56.00	28.00	25.C0	6.0C
7	0.0	0.0	0.0	0.0	C.0	0.0	79.00	42.0C	17.00
8	0.0	0.0	C.0	0.0	C.0	0.0	0.0	65.C0	34.0C

MATRIX OF EXPECTED VALUES -- ADULTS

1	43.00	2C.78	13.15	7.60	5.5S	3.25	2.44	1.55	0.64
2	0.0	36.00	11.53	6.67	4.9C	2.85	2.14	1.36	0.56
3	0.0	0.0	40.00	21.29	15.64	9.10	6.84	4.34	1.7S
4	0.0	0.0	0.0	34.00	13.28	7.72	5.81	3.68	1.51
5	0.0	0.0	0.0	0.0	46.00	12.37	9.30	5.SC	2.43
6	0.0	0.0	0.0	0.0	0.0	56.00	31.14	19.74	8.12
7	0.C	C.0	0.0	0.0	0.0	0.0	79.00	41.8C	17.20
8	0.0	0.0	0.0	0.0	C.0	0.0	0.0	65.00	34.00

MATRIX OF STANDARD NORMAL DEVIATES -- ADULTS

1	0.00	-0.17	-0.88	1.60	-1.1C	0.97	-0.28	1.17	-0.80
2	0.0	0.00	-1.36	0.91	C.5C	0.69	-0.1C	0.55	-0.75
3	C.0	C.0	0.00	0.60	C.86	-0.70	-0.71	-0.64	-0.59
4	0.0	0.0	0.0	0.00	-C.08	-0.26	-0.75	1.74	-0.42
5	0.0	0.0	0.0	0.0	0.0C	0.18	-0.76	1.28	-0.92
6	0.0	0.0	0.0	0.0	C.0	0.00	-0.57	1.2C	-0.75
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0C	0.03	-0.05
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0C	0.0C

MATRIX OF CATA VALUES -- YOUNG

1	132.00	48.00	33.00	13.00	8.00	9.00	8.00	5.0C	1.00
2	0.0	175.00	33.00	8.00	11.0C	10.00	4.00	7.00	1.00
3	0.0	C.0	165.00	39.00	23.00	12.00	13.00	5.00	7.00
4	0.0	C.0	0.0	193.00	51.0C	24.00	13.00	12.00	5.00
5	0.0	0.0	0.0	0.0	193.00	43.00	39.00	15.00	9.00
6	0.0	0.0	0.0	0.0	C.0	367.00	113.00	56.0C	32.00
7	0.0	0.0	0.0	0.0	C.0	0.0	392.00	176.0C	70.0C
8	0.0	C.0	0.0	0.0	C.0	0.0	0.0	342.0C	101.00

MATRIX OF EXPECTED VALUES -- YOUNG

1	132.00	47.22	29.89	17.28	12.7C	7.38	5.55	3.52	1.45
2	0.0	175.00	28.44	16.44	12.08	7.03	5.29	3.35	1.38
3	0.0	0.0	165.00	35.72	26.25	15.27	11.48	7.28	3.00
4	0.0	C.0	0.0	1S3.00	43.56	25.33	19.05	12.08	4.97
5	0.0	C.0	0.0	0.0	193.00	43.70	32.87	20.84	8.58
6	0.0	0.0	0.0	0.0	0.0	367.00	106.07	67.26	27.67
7	0.0	C.0	C.0	0.0	C.0	0.0	392.00	174.2S	71.71
8	0.0	C.0	0.0	0.0	C.0	0.0	0.0	342.CC	101.00

MATRIX OF STANCARD NORMAL DEVIATES -- YOUNG

1	0.0	C.11	0.57	-1.04	-1.32	0.60	1.04	0.7S	-0.37
2	0.0	0.0	0.86	-2.09	-C.31	1.12	-0.56	2.0C	-0.32
3	0.0	0.0	0.0	0.55	-C.64	-0.84	0.45	-0.85	2.32
4	0.0	C.0	0.0	0.0	1.14	-0.27	-1.39	-0.02	0.01
5	0.0	C.0	0.0	0.0	0.0	-0.11	1.08	-1.29	0.14
6	0.0	0.0	0.0	0.0	0.0	0.0	0.68	-1.3S	0.83
7	0.0	0.0	0.0	0.0	C.0	0.0	0.0	0.13	-0.2C
8	0.0	0.0	0.0	0.0	C.0	0.0	0.0	0.0	0.0C

Example 3.6e

YOUNG AND ADULT MALE MALLARDS BANDED PRESEASON IN SOUTHERN ONTARIO, 1965 34350

CHI-SQUARE TEST OF H1 VS H2

	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
I= 2	36	30	
	68	112	5.564
I= 3	40	59	
	83	133	0.112
I= 4	34	32	
	105	186	5.389
I= 5	46	30	
	134	189	9.008
I= 6	56	59	
	134	191	1.929
I= 7	79	59	
	238	213	0.851
I= 8	65	34	
	367	151	1.068

TOTAL CHI-SQUARE WITH 7 DEGREES OF FREEDOM = 23.920

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 23.92 = 0.00118

THIS TEST OF THE HYPOTHESIS H1 AGAINST THE HYPOTHESIS H2 TESTS THE ASSUMPTION THAT RECOVERY RATES FOR NEWLY RELEASED ADULTS ARE THE SAME AS FOR SURVIVORS OF PREVIOUSLY BANDED COHORTS.

CHI-SQUARE TEST OF H2 VS H3

TOTAL CHI-SQUARE WITH 7 DEGREES OF FREEDOM = 7.676

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 7.68 = 0.36205

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H0

TOTAL CHI-SQUARE 305.35 WITH 66 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 305.35 = 0.0

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H1

TOTAL CHI-SQUARE 68.67 WITH 50 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 68.67 = 0.04094

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H2

TOTAL CHI-SQUARE 44.75 WITH 43 D.F.

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 44.75 = 0.39820

FROM THE MODELS ABOVE, ONE SHOULD CHOOSE THE SIMPLEST MODEL (FEWEST PARAMETERS) THAT ADEQUATELY DESCRIBES THE DATA. ADEQUACY MAY BE JUDGED BY EXAMINING THE RESULTS OF (1) THE GOODNESS OF FIT TESTS, AND (2) THE TESTS BETWEEN SPECIFIC MODELS. FREQUENTLY, HO2 OR H1 IS ADEQUATE.

An Example

Example 3.6 serves as an illustration of a situation when the model under H_2 seems appropriate. The data are from a study on male mallards banded preseason in Southern Ontario 1965-72.

Example 3.6e shows the goodness of fit tests result in rejection of the models under both H_0 and H_1. Also, the test of H_1 vs. H_2, with a chi-square value of 23.92 and 7 df, is highly significant, with several of the individual chi-square components significantly large. Thus the H_1 model must be rejected in favor of H_2. Comparison of H_2 with H_3 does not suggest preference for H_3 so H_2 seems appropriate for these data.

Examination of the estimates of recovery rates of adults under the H_2 model indicates that first-year recovery rates (f_i''') are significantly higher than adult recovery rates in latter years (f_i). This is probably the result of band solicitation by conservation agencies in Ontario.

3.8 Summary of the Models Proposed

The models of this chapter have proved to be useful in the analysis of data from banding studies involving both young and adults of many waterfowl species. Some general conclusions can be made on the basis of these results.

The H_0 and H_{01} models are too restrictive, i.e., young and adults do have different survival and recovery rates, and recovery rates differ from year to year.

Frequently either the H_{02} or the H_1 model seems to be adequate (Examples 3.2 and 3.5). The H_1 model has the practical advantage in that estimates are easily computed with a desk calculator for small data sets.

Occasionally, as in Example 3.6, there appears to be a real difference in the recovery rate between newly released adults and that for survivors of previous releases. In this case the model under H_2 should be used although for values of N_i and M_i usually encountered in practice, the H_2 estimators will have poor precision.

In-season banding may give rise to a situation where H_2 appears to be appropriate, but in fact is not. Sometimes, however, H_2 may be a reasonable approximation when in-season banding takes place (see Example 3.4).

The H_3 model appears to be unnecessarily general for data sets we have analyzed and subadults do not have very different survival and recovery rates from those of older birds.

We conclude this chapter with a discussion which demonstrates the futility of banding only young when we have reason to believe that survival and recovery rates are different for young and adults.

3.9 Why Survival Rates Cannot be Estimated if Only Young are Banded

For many species the banding of young is much easier to accomplish than the banding of adults. For this reason there is strong temptation to concentrate the banding effort exclusively upon young; this is a pointless practice. Analysis of numerous data sets for birds has shown that the survival rate of young is typically lower than adult survival rates $(S_i' < S_i)$. Also first-year band recovery rates for young are typically higher than for adults $(f_i' > f_i)$. Consequently, an appropriate model for band recoveries from young is the H_1 model. Specifically its structure is shown below for $k = \ell = 4$.

Number banded	Year of recovery 1	2	3	4
M_1	$M_1 f_1'$	$M_1 S_1' f_2$	$M_1 S_1' S_2 f_3$	$M_1 S_1' S_2 S_3 f_4$
M_2		$M_2 f_2'$	$M_2 S_2' f_3$	$M_2 S_2' S_3 f_4$
M_3			$M_3 f_3'$	$M_3 S_3' f_4$
M_4				$M_4 f_4'$

From banding only young, the direct recovery rates f_i' can be estimated, but without additional information (bandings of adults) the survival rates S_i' of young can never be estimated. The impossibility of estimating S_i' persists no matter what simplifying assumptions are made about how rates vary over time. For example, assuming $S_i \equiv S$, $i = 1, \ldots, \ell - 1$ (constant adult survival rate) does not allow estimability. Assuming constant adult recovery rates $(f_i \equiv f,\ i = 2, \ldots, \ell)$ renders adult survival rates S_i estimable from young only recoveries, but leaves S_i' and f confounded. Moreover, this assumption is certainly wrong. In the extreme case of assuming no parameter varies over time the model structure depends upon only four parameters (f', f, S', S):

Number banded	Year of recovery 1	2	3	4
M_1	M_1f'	$M_1S'f$	$M_1S'Sf$	$M_1S'SSf$
M_2		M_2f'	$M_2S'f$	$M_2S'Sf$
M_3			M_3f'	$M_3S'f$
M_4				M_4f'

This is the structure for young under the H_{01} model.

Note that the parameters S' and f always occur together as a product. This is why, even in this simple model, these two parameters cannot be separately estimated; only the product $S'f$ can be estimated if no adults are banded. To illustrate this point we could define "new" parameters as $S_o' = cS'$ and $f_o = \frac{1}{c} f$ for infinitely many values of the constant c, and the product $S'f$ is not changed. Thus, the expected cell probabilities cannot be given as a product of unique parameters S' and f; for example

$$\mathbf{E}(Q_{12}) = M_1 S' f = M_1 (S'c) (\frac{1}{c} f) = M_1 S_o' f_o ,$$

where S' and S_o' can be quite different. In statistical terms this parameterization of the band recovery model leaves S' and f nonidentifiable.

This nonidentifiability of S_i' persists in the H_1 model for banding of young only. In fact, not even S_i or f_i is estimable under the H_1 model if only young are banded. This problem may be illustrated numerically by choosing several different configurations of annual survival and recovery rates which produce exactly the same array of expected band recoveries. For $k = \ell = 4$, suppose a banding experiment on only young birds produced the following recoveries:

Number banded	Year of recovery 1	2	3	4
10,000	1,000	500	150	168
5,000		1,000	100	112
9,000			1,800	216
8,000				1,200

(symbolic expectations under H_1 were given above). The direct recovery rates (the f_i') are unique. For this example $f_1' = 0.1$, $f_2' = 0.2$, $f_3' = 0.2$, and $f_4' = 0.15$. The remaining parameters need not be unique to produce this expected data array. In fact we used $S_2 = 0.6$ and $S_3 = 0.7$. But for the choice of S_i' and f_i' the following three parameter sets will all produce these exact same expected recovery data:

Parameter set	S_1'	S_2'	S_3'	f_2	f_3	f_4
A	0.5	0.4	0.3	0.1	0.05	0.08
B	0.25	0.2	0.15	0.2	0.1	0.16
C	0.625	0.5	0.375	0.08	0.04	0.064

For example, consider the expected recoveries from the first banded cohort in the fourth recovery year:

Parameter set	$\mathbf{E}(Q_{14}) = M_1 S_1' S_2 S_3 f_4$
A	10,000 (.5) (.6) (.7) (.08) = 168
B	10,000 (.25) (.6) (.7) (.16) = 168
C	10,000 (.625) (.6) (.7) (.064) = 168

The reader should verify that these three parameter sets give exactly the same expected band recovery data. Consequently, it is impossible to infer from these "data" which parameter set was used to generate these data.

It is possible to test for first-year age effects in banding data for young. This is a test of whether or not Model 1 (Chapter 2) fits the data and is discussed in Section 2.6 as a test of the null hypothesis that Model 1 fits the data vs. the alternative that Model 0 fits. In fact Model 0 cannot be distinguished by any statistical test from the model for recovery data of young under H_1 (see Section 2.5, *Proper and Improper Use of Model 0).*

For the array of expected band recoveries above, the test is based on the two contingency tables below, the elements of which come from the rows and columns of the "data" array as explained in Robson and Youngs (1971) (also see Section 2.6 for an explanation of this test):

rows	columns 2	columns 3+4
1	500	318
2	1,000	212

rows	columns 3	columns 4
1+2	250	280
3	1,800	216

The computed chi-squares, each with 1 df, are 115.8 and 474.6, respectively. We would thus conclude age-specific recovery and/or survival rates occur in the population. If these were adult bandings, we might assume Model 0 was the true model and thus proceed to get meaningful survival estimates. But when we know the recovery data came from bandings of young only, we must abandon all attempts to meaningfully estimate survival rates from these data.

It may be useful to heuristically show how S_i' can be estimated if adult recovery data are available. First

$$S_i'\rho_{i+1} = \mathbf{E}(Q_{i\cdot} - Q_{ii})/M_i$$

is clearly estimable from band recoveries of young, while

$$\rho_{i+1} = \mathbf{E}(R_{i+1\cdot})/N_{i+1}$$

is estimable from band recoveries of adults ($\rho_{i+1} = f_{i+1} + S_{i+1}f_{i+2} + \ldots + S_{i+1}\cdots S_{\ell-1}f_\ell$). An estimator of S_i' (in fact the ML estimator) is

$$\hat{S}_i' = \frac{\widehat{S_i'\rho_{i+1}}}{\hat{\rho}_{i+1}} = \frac{Q_{i\cdot} - Q_{ii}}{M_i} \bigg/ \frac{R_{i+1\cdot}}{N_{i+1}} .$$

Without adult data, ρ_{i+1} cannot be estimated.

The above discussion concerning the estimation of survival rates from birds banded as young assumes the models of Chapters 2 and 3. The astute reader might ask if there exist models for band recovery data allowing the estimation of survival rates from banding only young birds. The model underlying the composite dynamic life table method is such a model (cf. Hickey 1952; Seber 1972; Anderson and Burnham 1976). Seber (1971) has considered the proper statistical analysis of data given this model structure. The composite dynamic model assumes that both recovery and survival rates are age-dependent only; i.e., these rates are unaffected by year-to-year changes in hunting regulations, habitat, weather, etc.

A second assumption of the composite dynamic model is that the age-specific recovery rate is a constant fraction of the age-specific mortality rate. For the two-age-class case (i.e., young and adults) this means we must assume

$$\frac{f_i'}{1 - S_i'} = \frac{f_i}{1 - S_i} = c \qquad , i = 1, \ldots, \ell - 1$$

where c is a constant.

Both assumptions are critical and both are demonstrably invalid for banding studies of game birds. Detailed analysis of mallard banding data has documented the untenability of both assumptions (Anderson 1975). Moreover, the estimates of age-specific survival rates computed assuming this model (i.e., the composite dynamic method) are severely biased when these assumptions fail.

It is our intention in this handbook to detail some correct models and methods for the analysis of band recovery data. Thus, we will not dwell on the composite dynamic method, nor the concomitant model under which young-only recovery data could be analyzed (e.g., Seber 1971). Firstly, the model is invalid for waterfowl (Burnham and Anderson 1979) and nongame species (Anderson et al. 1981). Secondly, serious bias occurs when these crucial assumptions are not met (Anderson et al. 1985). The interested reader can pursue this topic further by referring to reports by Anderson (1975) and Anderson and Burnham (1976).

Finally, we note that it is possible to construct yet other models that have age-specific survival rates in their structure, and are such that from banding young only these survival rates could be estimated *if the model were true*. However, such models are usually demonstrably invalid, on either theoretical and/or empirical grounds. For example, let survival be age-specific and let the recovery rate be constant, independent of age or calendar year (cf. Seber 1971). This model seems unreasonable: empirically we know f varies by calendar year and by age (young vs. adult), and theoretically it does not make sense to have age-dependent annual mortality but age-independent recovery rates (mortality is a "hidden" component of f). *We can not emphasize too strongly that, based on our current knowledge, there is no valid way to estimate age-specific survival rates from only the banding of young.*