Chapter 4. Models for Birds Banded as Young, Subadults, and Adults

4.1 Introduction

The models considered in this chapter permit estimation under the assumption that survival and recovery rates are age-dependent for the first 2 years of life. The experimental situation requires that members of the resulting three age classes be recognizable at the time of banding because the numbers banded and subsequent recoveries must be recorded separately for each age class.

We refer to the three age classes as young (birds in their first year), subadults (birds in their second year), and adults (birds over 2 years old). This terminology is commonly used in relation to certain species (e.g., geese) where 2-year-olds are not sexually mature, and it should not be confused with the use of the terms "young" and "adults" in Chapters 2 and 3.

The methods of this chapter are of limited use in relation to waterfowl studies, because three age groups are recognizable in only a few species (e.g., geese). Also, the analysis of a considerable amount of waterfowl data by the methods of Chapter 3 has indicated that the age-dependence assumption, common to the models of this chapter and to H_3 of Chapter 3, is unnecessarily general for many species. However, several sets of goose data should be collected and analyzed using the methods of this chapter to decide whether they are entirely inappropriate for waterfowl. We suggest that these methods may be of use in studies on fish populations where age may sometimes be easily determined by length, or, if necessary, by scale analysis and where the age-dependence assumption may not be too general.

The FORTRAN program BROWNIE is available to facilitate the use of these methods and sample output from the program appears in the examples. Data used in these examples are not real but have been generated stochastically.

Because the models of this chapter apply to a more specific situation and hence are likely to be less useful than their Chapter 3 counterparts, the development and treatment of these models is less extensive than those in Chapter 3. Also less detail is given in describing methods of estimation and testing procedures when analogous procedures in Chapter 3 can be referred to. (For a more detailed mathematical treatment see Brownie 1973). Similarly, the output of the FORTRAN program is much briefer than that for the two age-class models of Chapter 3. The reader should not attempt to use the models in Chapter 4 without first understanding Chapter 3.

Experimental Situation

The number banded and subsequent recoveries are recorded separately for three groups, i.e., for birds banded as young, subadults, and adults. In all other respects the experimental situation is like that of Chapter 3 and further description is omitted. We emphasize that each of the three age classes must be represented in every cohort released or parameters of interest will not be estimable.

Notation and Definitions

k = the number of years at the start of which banded birds are released.
ℓ = the number of years during which recoveries are recorded, $\ell \geqslant k$.
$s = \ell - k$ = the number of years, beyond the year of the last release, when recoveries are recorded, $s \geqslant 0$.
N_i = the number of adults banded and released in year i, $i = 1, \ldots, k$.
K_i = the number of subadults banded and released in year i, $i = 1, \ldots, k$.
M_i = the number of young banded and released in year i, $i = 1, \ldots, k$.
R_{ij} = the number of bands recovered in year j from the adults released in year i, $i = 1, \ldots, k$, $j = i, \ldots, \ell$.
Y_{ij} = the number of bands recovered in year j from the subadults released in year i, $i = 1, \ldots, k$, $j = i, \ldots, \ell$.
Q_{ij} = the number of bands recovered in year j from the young released in year i, $i = 1, \ldots, k$, $j = i, \ldots, \ell$.

The data are presented as in Chapter 3 except that three data arrays are present instead of two. This is illustrated in Table 4.1 in terms of N_i, K_i, M_i, R_{ij}, Y_{ij} and Q_{ij} for a banding study with 3 years of banding and 5 years of recovery ($k=3$, $\ell=5$, $s=2$). Some useful subtotals are indicated in the table.

Table 4.1. *Representation of the data for a banding study with $k=3$, $\ell=5$, and $s=2$ when young, subadults, and adults are banded and released each year.*

Year banded	Number banded	Year of recovery 1	2	3	4	$\ell=5$	Row totals
			Birds banded as adults				
1	N_1	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	$R_{1\cdot}=T_1$
2	N_2		R_{22}	R_{23}	R_{24}	R_{25}	$R_{2\cdot}$
$k=3$	N_3			R_{33}	R_{34}	R_{35}	$R_{3\cdot}$
	Column totals	$R_{\cdot 1}$	$R_{\cdot 2}$	$R_{\cdot 3}$	$R_{\cdot 4}$	$R_{\cdot 5}=T_5$	
			Birds banded as subadults				
1	K_1	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}	$Y_{i\cdot}=V_1$
2	K_2		Y_{22}	Y_{23}	Y_{24}	Y_{25}	$Y_{2\cdot}$
$k=3$	K_3			Y_{33}	Y_{34}	Y_{35}	$Y_{3\cdot}$
	Column totals	$Y_{\cdot 1}$	$Y_{\cdot 2}$	$Y_{\cdot 3}$	$Y_{\cdot 4}$	$Y_{\cdot 5}=V_5$	
			Birds banded as young				
1	M_1	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	$Q_{1\cdot}=U_1$
2	M_2		Q_{22}	Q_{23}	Q_{24}	Q_{25}	$Q_{2\cdot}$
$k=3$	M_3			Q_{33}	Q_{34}	Q_{35}	$Q_{3\cdot}$
	Column totals	$Q_{\cdot 1}$	$Q_{\cdot 2}$	$Q_{\cdot 3}$	$Q_{\cdot 4}$	$Q_{\cdot 5}=U_5$	

The arrays representing the recovery matrices are R_{ij}'s for adults, Y_{ij}'s for subadults, and Q_{ij}'s for young.

As before, row, column, and certain block totals of the recovery matrices are used in summarizing the data. The notation is similar to that of Chapter 3. Thus the row totals of the recovery matrices for adults, subadults, and young are denoted by $R_{i\cdot}$, $Y_{i\cdot}$, and $Q_{i\cdot}$, respectively, $i=1,\ldots,k$. Similarly, $R_{\cdot j}$, $Y_{\cdot j}$ and $Q_{\cdot j}$, $j=1,\ldots,\ell$ are the corresponding column totals, and

$$T_i=\begin{cases}R_{1\cdot} & ,i=1\\ R_{i\cdot}+T_{i-1}-R_{\cdot i-1} & ,i=2,\ldots,k,\end{cases}$$

$$V_i=\begin{cases}Y_{i\cdot} & ,i=1\\ Y_{i\cdot}+V_{i-1}-Y_{\cdot i-1} & ,i=2,\ldots,k,\end{cases}$$

$$U_i=\begin{cases}Q_{i\cdot} & ,i=1\\ Q_{i\cdot}+U_{i-1}-Q_{\cdot i-1} & ,i=2,\ldots,k,\end{cases}$$

are corresponding block totals.

Subtotals involving elements from all three recovery matrices are

$$A_i=\begin{cases}R_{\cdot 1} & ,i=1\\ R_{\cdot i}+Y_{\cdot i}+Q_{\cdot i}-Y_{ii}-Q_{ii}-Q_{i-1,i} & ,i=2,\ldots,k.\end{cases}$$

where A_i is the total number of recoveries from adults in year i (including recoveries from previously released young and subadults which have survived to adulthood),

$$D_i = \begin{cases} T_1 = R_{1\cdot} & , i=1 \\ T_i + V_i + U_i - Y_{i\cdot} - Q_{i\cdot} - Q_{i-1\cdot} + Q_{i-1,i-1} & , i=2,\ldots,k. \end{cases}$$

Synthetic data for a banding study with $k=\ell=6$ are presented in Table 4.2 and are used to illustrate the calculation of the above subtotals.

Table 4.2 *Synthetic data for a study with* $k=\ell=6, s=0$.

Year banded	Number banded	Year of recovery 1	2	3	4	5	6
				Birds Banded as adults			
1	700	36	20	26	11	17	6
2	700		23	31	22	25	16
3	700			42	24	23	11
4	700				38	37	20
5	700					38	19
6	700						23
				Birds banded as subadults			
1	800	80	12	20	12	19	3
2	800		57	33	17	34	7
3	800			83	26	25	13
4	800				64	25	14
5	800					115	26
6	800						57
				Birds banded as young			
1	1,000	161	34	21	7	18	4
2	1,000		109	73	12	16	7
3	1,000			171	51	26	13
4	1,000				157	57	10
5	1,000					198	39
6	1,000						102

Subtotals

i	$\mathring{R}_{i\cdot}$	$R_{\cdot i}$	$Y_{i\cdot}$	$Y_{\cdot i}$	$Q_{i\cdot}$	$Q_{\cdot i}$	T_i	V_i	U_i	A_i	D_i
1	116	36	146	80	245	161	116	146	245	36	116
2	117	43	148	69	217	143	197	214	301	55	263
3	100	99	147	136	261	265	254	292	419	173	449
4	95	95	103	119	224	227	250	259	378	169	470
5	57	140	141	218	237	315	212	281	388	303	436
6	23	95	57	120	102	175	95	120	175	192	192

Parameters and Assumptions

The models of this chapter are closely analogous to the models in Chapter 3 and are constructed in terms of the same basic parameters: S, the annual survival rate and f, the annual recovery rate. In all three models f and S are assumed to be year-specific (indicated as usual by subscripts, e.g., f_i) and age-dependent for the three classes—young, subadult and adult (indicated by superscript primes).

Again we note that the assumption that banded birds in the population suffer independent fates is questionable for species such as geese where young and adults tend to migrate and winter in distinct family groups. This point is particularly important in this chapter because several species of geese are among the few waterfowl species in which young, subadults, and adults are distinguishable through examination in the field. Failure of this independence assumption will not bias the estimates of survival or recovery rates but will invalidate the sampling variances of estimators.

4.2 The Model Under H_4

We begin with the model which is the analogue of the H_1 model of Chapter 3, and which we call the model under H_4 (or the H_4 model). This is the simplest of the three models of this chapter and the assumptions of H_4 are:

(1) Annual survival and recovery rates are year-specific;
(2) annual survival and recovery rates are different for young, subadults, and adults; and
(3) reporting rates are not dependent on the time of release.

The parameters are:

f_i'' = recovery rate in year i for young banded in year i, $i = 1, \ldots, k$,

S_i'' = survival rate in year i for young banded in year i, $i = \begin{cases} 1, \ldots, k-1, & \text{if } \ell = k \\ 1, \ldots, k, & \text{if } \ell > k, \end{cases}$

f_i' = recovery rate in year i for subadults banded in year i or survivors of young banded in year $i-1$, $i = 1, \ldots, k$,

S_i' = survival rate in year i for subadults banded in year i or survivors of young banded in year $i-1$, $i = \begin{cases} 1, \ldots, k-1, & \text{if } \ell = k \\ 1, \ldots, k, & \text{if } \ell > k, \end{cases}$

f_i = recovery rate in year i for adults, $i = 1, \ldots, \ell$,
S_i = survival rate in year i for adults, $i = 1, \ldots, \ell - 1$.

The structure of the model is expressed by the expected numbers of band recoveries in terms of N_i, K_i, M_i, f_i, f_i', f'', S_i, S_i' and S'' as shown in Table 4.3.

Table 4.3. *Expected numbers of band recoveries under H_4 for a banding study with $k = 3, \ell = 5, s = 2$.*

Year banded	Number banded	Year of recovery: 1	2	3	4	5
		Birds banded and released as adults				
1	N_1	$N_1 f_1$	$N_1 S_1 f_2$	$N_1 S_1 S_2 f_3$	$N_1 S_1 S_2 S_3 f_4$	$N_1 S_1 S_2 S_3 S_4 f_5$
2	N_2		$N_2 f_2$	$N_2 S_2 f_3$	$N_2 S_2 S_3 f_4$	$N_2 S_2 S_3 S_4 f_5$
3	N_3			$N_3 f_3$	$N_3 S_3 f_4$	$N_3 S_3 S_4 f_5$
		Birds banded and released as subadults				
1	K_1	$K_1 f_1'$	$K_1 S_1' f_2$	$K_1 S_1' S_2 f_3$	$K_1 S_1' S_2 S_3 f_4$	$K_1 S_1' S_2 S_3 S_4 f_5$
2	K_2		$K_2 f_2'$	$K_2 S_2' f_3$	$K_2 S_2' S_3 f_4$	$K_2 S_2' S_3 S_4 f_5$
3	K_3			$K_3 f_3'$	$K_3 S_3' f_4$	$K_3 S_3' S_4 f_5$
		Birds banded and released as young				
1	M_1	$M_1 f_1''$	$M_1 S_1'' f_2'$	$M_1 S_1'' S_2' f_3$	$M_1 S_1'' S_2' S_3 f_4$	$M_1 S_1'' S_2' S_3 S_4 f_5$
2	M_2		$M_2 f_2''$	$M_2 S_2'' f_3'$	$M_2 S_2'' S_3' f_4$	$M_2 S_2'' S_3' S_4 f_5$
3	M_3			$M_3 f_3''$	$M_3 S_3'' f_4'$	$M_3 S_3'' S_4' f_5$

Estimation of Parameters

ML estimators of the annual recovery rates are:

$$\hat{f}_i = \frac{R_{i\cdot}}{N_i} \frac{A_i}{D_i} \qquad , i = 1, \ldots, k,$$

$$\hat{f}_i' = \begin{cases} \dfrac{Y_{11}}{K_1} & , i=1 \\ \dfrac{Y_{i\cdot}}{K_i} \dfrac{Y_{ii}+Q_{i-1,i}}{Y_{i\cdot}+Q_{i-1\cdot}-Q_{i-1,i-1}} & , i=2,\ldots,k, \end{cases}$$

$$\hat{f}_i'' = \frac{Q_{ii}}{M_i} \qquad , i=1,\ldots,k.$$

The calculation of estimates is illustrated for the data in Table 4.2 and the results can be compared with the printout of Example 4.1b,

$$\hat{f}_1 = \frac{R_{1\cdot} \times A_1}{N_1 \times D_1} = \frac{116 \times 36}{700 \times 116} = 0.0514,$$

$$\hat{f}_2 = \frac{R_{2\cdot} \times A_2}{N_2 \times D_2} = \frac{117 \times 55}{700 \times 263} = 0.0350,$$

$$\hat{f}_1' = \frac{Y_{11}}{K_1} = \frac{80}{800} = 0.1000,$$

$$\hat{f}_2' = \frac{Y_{2\cdot} \times (Y_{22}+Q_{12})}{K_2 \times (Y_{2\cdot}+Q_{1\cdot}-Q_{11})} = \frac{148 \times 91}{800 \times 232} = 0.0726,$$

$$\hat{f}_1'' = \frac{Q_{11}}{M_1} = \frac{161}{1{,}000} = 0.1610,$$

$$\hat{f}_2'' = \frac{Q_{22}}{M_2} = \frac{109}{1{,}000} = 0.1090.$$

ML estimators of annual survival rates are:

$$\hat{S}_i = \frac{R_{i\cdot}}{N_i} \frac{D_i - A_i}{D_i} \frac{N_{i+1}}{R_{i+1\cdot}} \qquad , i=1,\ldots,k-1$$

$$\hat{S}_i' = \begin{cases} \dfrac{Y_{1\cdot}-Y_{11}}{K_1} \dfrac{N_2}{R_{2\cdot}} & , i=1 \\ \dfrac{Y_{i\cdot}}{K_i} \left(1 - \dfrac{Y_{ii}+Q_{i-1,i}}{Y_{i\cdot}+Q_{i-1\cdot}-Q_{i-1,i-1}}\right) \dfrac{N_{i+1}}{R_{i+1\cdot}} & , i=2,\ldots,k-1, \end{cases}$$

$$\hat{S}_i'' = \frac{Q_{i\cdot}-Q_{ii}}{M_i} \frac{K_{i+1}}{Y_{i+1\cdot}} \qquad , i=1,\ldots,k-1.$$

For example,

$$\hat{S}_1 = \frac{R_{1\cdot} \times (D_1 - A_1) \times N_2}{N_1 \times D_1 \times R_{2\cdot}} = \frac{116 \times (116-36) \times 700}{700 \times 116 \times 117} = 0.6838,$$

$$\hat{S}_2 = \frac{R_{2\cdot} \times (D_2 - A_2) \times N_3}{N_2 \times D_2 \times R_{3\cdot}} = \frac{117 \times (263-55) \times 700}{700 \times 263 \times 100} = 0.9253,$$

$$\hat{S}_1' = \frac{(Y_{1\cdot}-Y_{11}) \times N_2}{K_1 \times R_{2\cdot}} = \frac{66 \times 700}{800 \times 117} = 0.4936,$$

$$\hat{S}_2' = \frac{Y_{2\cdot}}{K_2} \left(1 - \frac{Y_{22}+Q_{12}}{Y_{2\cdot}+Q_{1\cdot}-Q_{11}}\right) \frac{N_3}{R_{3\cdot}} = \frac{148}{800} \left(1 - \frac{91}{232}\right) \frac{700}{100} = 0.7870,$$

$$\hat{S}_1'' = \frac{(Q_{1\cdot}-Q_{11}) \times K_2}{M_1 \times Y_{2\cdot}} = \frac{84 \times 800}{1{,}000 \times 148} = 0.4541,$$

$$\hat{S}_2'' = \frac{(Q_{2\cdot}-Q_{22}) \times K_3}{M_2 \times Y_{3\cdot}} = \frac{108 \times 800}{1{,}000 \times 147} = 0.5878.$$

It is easily seen that these calculations agree with the results in the output from program BROWNIE in Example 4.1b.

Bias-adjusted ML estimators of survival (analogous to the H_1 estimators $\tilde{S}_i$ and $\tilde{S}'_i$) are easily constructed but are not defined here. We repeat that for sample sizes which are large enough that the precision of estimators is good (and hence estimates are reliable) the difference between the adjusted and unadjusted estimators will be negligible.

If $\ell > k$, the parameters $S_k, \ldots, S_{\ell-1}, S'_k, S''_k, f_{k+1}, \ldots, f_\ell$ are not separately estimable.

Sampling Variances, Standard Errors, and Confidence Intervals

As in Chapter 3, estimates of the sampling variances and 95% confidence intervals are obtained to provide an indication of the precision of the ML estimates. The notation used to define the variance estimators is like that in Chapter 3, and these are used to obtain estimates of standard errors and confidence intervals as in Chapter 3. The required variance estimators are listed below, followed by a single numerical illustration of the computational procedure.

$$\text{var}(\hat{f}_i) = (\hat{f}_i)^2 \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} + \frac{1}{A_i} - \frac{1}{D_i}\right] \qquad , i = 1, \ldots, k,$$

$$\text{var}(\hat{S}_i) = (\hat{S}_i)^2 \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} + \frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}} + \frac{1}{D_i - A_i} - \frac{1}{D_i}\right] \qquad , i = 1, \ldots, k-1,$$

$$\text{var}(\hat{f}'_i) = \begin{cases} \hat{f}'_1 (1 - \hat{f}'_1) / K_1 & , i = 1 \\ (\hat{f}'_i)^2 \left[\frac{1}{Y_{i\cdot}} - \frac{1}{K_i} + \frac{1}{Y_{ii} + Q_{i-1,i}} - \frac{1}{Y_{i\cdot} + Q_{i-1\cdot} - Q_{i-1,i-1}}\right] & , i = 2, \ldots, k, \end{cases}$$

$$\text{var}(\hat{S}'_i) = \begin{cases} (\hat{S}'_1)^2 \left[\frac{1}{Y_{1\cdot} - Y_{11}} - \frac{1}{K_1} + \frac{1}{R_{2\cdot}} - \frac{1}{N_2}\right] & , i = 1 \\ (\hat{S}'_i)^2 \left[\frac{1}{Y_{i\cdot}} - \frac{1}{K_i} + \frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}} + \frac{1}{Y_{i\cdot} - Y_{ii} + Q_{i-1\cdot} - Q_{i-1,i-1} - Q_{i-1,i}} - \frac{1}{Y_{i\cdot} + Q_{i-1\cdot} - Q_{i-1,i-1}}\right] & , i = 2, \ldots, k-1, \end{cases}$$

$$\text{var}(\hat{f}''_i) = \hat{f}''_i (1 - \hat{f}''_i) / M_i \qquad , i = 1, \ldots, k,$$

$$\text{var}(\hat{S}''_i) = (\hat{S}''_i)^2 \left[\frac{1}{Q_{i\cdot} - Q_{ii}} - \frac{1}{M_i} + \frac{1}{Y_{i+1\cdot}} - \frac{1}{K_{i+1}}\right] \qquad , i = 1, \ldots, k-1.$$

For the data of Table 4.2,

$$\text{var}(\hat{S}_2) = (\hat{S}_2)^2 \left[\frac{1}{R_{2\cdot}} - \frac{1}{N_2} + \frac{1}{R_{3\cdot}} - \frac{1}{N_3} + \frac{1}{D_2 - A_2} - \frac{1}{D_2}\right] = (0.9253)^2 \left[\frac{1}{117} - \frac{1}{700} + \frac{1}{100} - \frac{1}{700} + \frac{1}{208} - \frac{1}{263}\right] = 0.01429416,$$

$$\text{se}(\hat{S}_2) = \sqrt{0.01429416} = 0.1196,$$

and $1.46 \times \text{se}(\hat{S}_2) = 0.2344$, thus the 95% confidence interval for S_2 is (0.6909, 1.1597).

Comparison with the output of Example 4.1b shows that the lower bound of the confidence interval in the example is slightly different from the above result. As mentioned before, such differences are due to the greater accuracy of the calculations performed by the computer. Note also that the confidence interval for S_2 is wide and contains impossible values. In real data this would indicate an insufficient number of birds were banded. The output in Example 4.1b shows that estimated standard errors and confidence intervals are printed beside each estimate so that tedious computations can be avoided by using the FORTRAN program BROWNIE.

Sampling Covariances and Correlations

Estimates of the sampling covariances and correlations between the ML estimators are obtained as in Chapter 3, and the notation used is again similar. All the covariance estimates listed below (and the corresponding correlations) are contained in the output of the FORTRAN program (see Example 4.1c) and are presented here largely for reference purposes.

Again we point out that these are estimates of the sampling covariances and correlations between the ML estimators and do not reflect a relationship between the unknown parameters. It is important to obtain an idea of the magnitudes of the correlations, because if these are substantial, they will obscure, or be confounded with, any relationship that exists between the unknown parameters. In Example 4.1c we see that many of these correlations are estimated to be quite large. Nonzero sampling covariances are estimated by

$$\text{cov}(\hat{f}_i, \hat{S}_i) = \hat{f}_i \hat{S}_i \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} - \frac{1}{D_i}\right] \qquad , i = 1, \ldots, k-1,$$

$$\text{cov}(\hat{f}_{i+1}, \hat{S}_i) = -\hat{f}_{i+1} \hat{S}_i \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad , i = 1, \ldots, k-1,$$

$$\text{cov}(\hat{f}_{i+1}, \hat{S}'_i) = -\hat{f}_{i+1} \hat{S}'_i \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad , i = 1, \ldots, k-1,$$

$$\text{cov}(\hat{S}_i, \hat{S}_{i+1}) = -\hat{S}_i \hat{S}_{i+1} \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad , i = 1, \ldots, k-2,$$

$$\text{cov}(\hat{S}_i, \hat{S}'_i) = \hat{S}_i \hat{S}'_i \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad , i = 1, \ldots, k-1,$$

$$\text{cov}(\hat{S}_{i+1}, \hat{S}'_i) = -\hat{S}_{i+1} \hat{S}'_i \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \qquad , i = 1, \ldots, k-2,$$

$$\text{cov}(\hat{f}'_i, \hat{S}'_i) = \begin{cases} -\hat{f}'_1 \hat{S}'_1 / K_1 & , i = 1 \\ \hat{f}'_i \hat{S}'_i \left[\frac{1}{Y_{i+1\cdot}} - \frac{1}{K_{i+1}}\right] & , i = 2, \ldots, k-1, \end{cases}$$

$$\text{cov}(\hat{f}'_{i+1}, \hat{S}''_i) = -\hat{f}'_{i+1} \hat{S}''_i \left[\frac{1}{Y_{i+1\cdot}} - \frac{1}{K_{i+1}}\right] \qquad , i = 1, \ldots, k-1,$$

$$\text{cov}(\hat{S}'_{i+1}, \hat{S}''_i) = -\hat{S}'_{i+1} \hat{S}''_i \left[\frac{1}{Y_{i+1\cdot}} - \frac{1}{K_{i+1}}\right] \qquad , i = 1, \ldots, k-2,$$

$$\text{cov}(\hat{f}''_i, \hat{S}''_i) = -\hat{f}''_i \hat{S}''_i / M_i \qquad , i = 1, \ldots, k-1.$$

Goodness of Fit Test

As in Chapter 3, a goodness of fit test is computed for each model to help in judging the adequacy of each model and in choosing the best model for a given data set. The goodness of fit test of the model under H_4 is computed in program BROWNIE in a manner analogous to the computation of the goodness of fit test of the model under H_1. As before, the test statistic is chi-square distributed if the assumptions of H_4 hold, and the result is interpreted in the usual way (i.e., "large" chi-square values indicate that agreement between model and data is poor).

The test for the synthetic data of Table 4.2 (see Example 4.1h) gives a chi-square value of 26.43 with 3C df. The probability under H_4 of a value at least this large is 0.653 and hence there is no reason to suspect that the assumptions of H_4 are incorrect. (This is not surprising because the data were generated using the model under H_4 as the probability model.)

4.3 The Model Under H_6

We now consider the model which is the analog of the H_2 model of Chapter 3. The model under H_6 is the most general of the three models of this chapter. The intermediate model (the model under H_5) is discussed in the next section for reasons given there. The assumptions of H_6 are:

(1) Annual survival and recovery rates are year-specific;
(2) annual survival and recovery rates are different for young, subadults, and adults; and
(3) in any year, the reporting rate for new releases is different from that for survivors of previous releases.

Assumptions 1 and 3 of H_6 are the same as those for H_2, and H_6 is a generalization of H_4 in the same way that H_2 is a generalization of H_1. Comments related to choosing between H_1 and H_2 (see Section 3.7) thus apply to choosing between H_4 and H_6.

Because of assumption 3, newly released adults and adult survivors from earlier releases have different recovery rates, defined as f_i'''' and f_i, respectively. Also, newly released subadults and the subadult survivors of young released the year before have different recovery rates, defined as f_i' and f_i''', respectively. All other parameters, i.e., S_i, S_i', f_i'', S_i'', are as defined for H_4.

The structure of the model is expressed in Table 4.4.

Table 4.4. *Expected numbers of band recoveries under H_6 for a banding study with $k=3$, $\ell=5$, $s=2$.*

Year banded	Number banded	Year of recovery 1	2	3	4	5
				Birds banded and released as adults		
1	N_1	N_1f_1''''	$N_1S_1f_2$	$N_1S_1S_2f_3$	$N_1S_1S_2S_3f_4$	$N_1S_1S_2S_3S_4f_5$
2	N_2		N_2f_2''''	$N_2S_2f_3$	$N_2S_2S_3f_4$	$N_2S_2S_3S_4f_5$
3	N_3			N_3f_3''''	$N_3S_3f_4$	$N_3S_3S_4f_5$
				Birds banded and released as subadults		
1	K_1	K_1f_1'	$K_1S_1'f_2$	$K_1S_1'S_2f_3$	$K_1S_1'S_2S_3f_4$	$K_1S_1'S_2S_3S_4f_5$
2	K_2		K_2f_2'	$K_2S_2'f_3$	$K_2S_2'S_3f_4$	$K_2S_2'S_3S_4f_5$
3	K_3			K_3f_3'	$K_3S_3'f_4$	$K_3S_3'S_4f_5$
				Birds banded and released as young		
1	M_1	M_1f_1''	$M_1S_1''f_2'''$	$M_1S_1''S_2'f_3$	$M_1S_1''S_2'S_3f_4$	$M_1S_1''S_2'S_3S_4f_5$
2	M_2		M_2f_2''	$M_2S_2''f_3'''$	$M_2S_2''S_3'f_4$	$M_2S_2''S_3'S_4f_5$
3	M_3			M_3f_3''	$M_3S_3''f_4'''$	$M_3S_3''S_4'f_5$

Estimation of Parameters

ML estimators of the individually estimable parameters are:

$$\hat{f}_i = \frac{R_{i\cdot} - R_{ii}}{N_i} \frac{A_i - R_{ii}}{D_i - A_i - R_{i\cdot} + R_{ii}} \quad , i = \begin{cases} 2, \ldots, k-1 \text{ if } \ell = k \\ 2, \ldots, k \quad \text{ if } \ell > k, \end{cases}$$

$$\hat{f}_i'''' = \frac{R_{ii}}{N_i} \quad , i = 1, \ldots, k,$$

$$\hat{f}_i' = \frac{Y_{ii}}{K_i} \quad , i = 1, \ldots, k,$$

$$\hat{f}_i'' = \frac{Q_{ii}}{M_i} \quad , i = 1, \ldots, k,$$

$$\hat{f}_i''' = \frac{Q_{i-1,i}}{Q_{i-1\cdot} - Q_{i-1,i-1} - Q_{i-1,i}} \frac{Y_{i\cdot} - Y_{ii}}{K_i} \quad , i = \begin{cases} 2, \ldots, k-1 \text{ if } \ell = k \\ 2, \ldots, k \quad \text{ if } \ell > k, \end{cases}$$

$$\hat{S}_i = \frac{R_{i\cdot} - R_{ii}}{N_i} \frac{N_{i+1}}{R_{i+1\cdot} - R_{i+1,i+1}} \frac{D_{i+1} - A_{i+1} - R_{i+1\cdot} + R_{i+1,i+1}}{D_{i+1} - R_{i+1\cdot}} \quad , i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k, \end{cases}$$

$$\hat{S}_i' = \frac{Y_{i\cdot} - Y_{ii}}{K_i} \frac{N_{i+1}}{R_{i+1\cdot} - R_{i+1,i+1}} \frac{D_{i+1} - A_{i+1} - R_{i+1\cdot} + R_{i+1,i+1}}{D_{i+1} - R_{i+1\cdot}} \quad , i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k, \end{cases}$$

$$\hat{S}_i'' = \frac{Q_{i\cdot} - Q_{ii} - Q_{i,i+1}}{M_i} \frac{K_{i+1}}{Y_{i+1\cdot} - Y_{i+1,i+1}} \quad , i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k. \end{cases}$$

For example, for the data of Table 4.2,

$$\hat{S}_1 = \frac{80 \times 700 \times (263 - 55 - 117 + 23)}{700 \times 94 \times (263 - 117)} = 0.6645,$$

$$\hat{S}_1' = \frac{66 \times 700 \times (263 - 55 - 117 + 23)}{800 \times 94 \times (263 - 117)} = 0.4797,$$

$$\hat{S}_1'' = \frac{(245 - 161 - 34) \times 800}{1{,}000 \times (148 - 57)} = 0.4396.$$

The (unadjusted) ML estimators defined above are evaluated and printed out by the FORTRAN program (see Example 4.1f). Bias-adjusted estimators are not presented here for the reason given in Section 4.2.

Sampling Variances, Standard Errors, and Confidence Intervals

The output of the FORTRAN program contains an estimate of the standard error and corresponding 95% confidence interval beside each ML estimate evaluated. The standard errors and confidence intervals are estimated as in the previous section and earlier chapters using the variance estimators defined below.

$$\text{var}(\hat{f}_i) = (\hat{f}_i)^2 \left[\frac{1}{R_{i\cdot} - R_{ii}} - \frac{1}{N_i} + \frac{1}{D_i - A_i - R_{i\cdot} + R_{ii}} + \frac{1}{A_i - R_{ii}}\right] \qquad , i = \begin{cases} 2, \ldots, k-1 & \text{if } \ell = k \\ 2, \ldots, k & \text{if } \ell > k, \end{cases}$$

$$\text{var}(\hat{f}_i''') = \hat{f}_i'''(1 - \hat{f}_i''') / N_i \qquad , i = 1, \ldots, k,$$

$$\text{var}(\hat{f}_i') = \hat{f}_i'(1 - \hat{f}_i') / K_i \qquad , i = 1, \ldots, k,$$

$$\text{var}(\hat{f}_i'') = \hat{f}_i''(1 - \hat{f}_i'') / M_i \qquad , i = 1, \ldots, k,$$

$$\text{var}(\hat{f}_i'') = (\hat{f}_i'')^2 \left[\frac{1}{Y_{i\cdot} - Y_{ii}} - \frac{1}{K_i} + \frac{1}{Q_{i-1\cdot} - Q_{i-1,i-1} - Q_{i-1,i}} + \frac{1}{Q_{i-1,i}}\right] \qquad , i = \begin{cases} 2, \ldots, k-1 & \text{if } \ell = k \\ 2, \ldots, k & \text{if } \ell > k, \end{cases}$$

$$\text{var}(\hat{S}_i) = (\hat{S}_i)^2 \left[\frac{1}{R_{i\cdot} - R_{ii}} - \frac{1}{N_i} + \frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}} + \frac{1}{D_{i+1} - A_{i+1} - R_{i+1\cdot} + R_{i+1,i+1}} - \frac{1}{D_{i+1} - R_{i+1\cdot}}\right] \qquad , i = \begin{cases} 1, \ldots, k-2 & \text{if } \ell = k \\ 1, \ldots, k-1 & \text{if } \ell > k, \end{cases}$$

$$\text{var}(\hat{S}_i') = (\hat{S}_i')^2 \left[\frac{1}{Y_{i\cdot} - Y_{ii}} - \frac{1}{K_i} + \frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}} + \frac{1}{D_{i+1} - A_{i+1} - R_{i+1\cdot} + R_{i+1,i+1}} - \frac{1}{D_{i+1} - R_{i+1\cdot}}\right] \qquad , i = \begin{cases} 1, \ldots, k-2 & \text{if } \ell = k \\ 1, \ldots, k-1 & \text{if } \ell > k, \end{cases}$$

$$\text{var}(\hat{S}_i'') = (\hat{S}_i'')^2 \left[\frac{1}{Q_{i\cdot} - Q_{ii} - Q_{i,i+1}} - \frac{1}{M_i} + \frac{1}{Y_{i+1\cdot} - Y_{i+1,i+1}} - \frac{1}{K_{i+1}}\right] \qquad , i = \begin{cases} 1, \ldots, k-2 & \text{if } \ell = k \\ 1, \ldots, k-1 & \text{if } \ell > k. \end{cases}$$

For a numerical illustration of the computational procedure, see Section 4.2. Use of the FORTRAN program is recommended to avoid the tedious calculations involved.

Comparison of the appropriate portions of Example 4.1 (i.e., 4.1b and 4.1f) shows that confidence intervals based on the H_6 estimators are considerably larger than those based on the corresponding H_4 estimators. This is to be expected since H_6 is the more general model with a larger number of parameters to be estimated. As discussed in a similar context in Chapters 2 and 3 this must be taken into account when deciding which model to choose for a given data set.

Sampling Covariances and Correlations

Estimates of the sampling covariances and correlations between the ML estimators are contained in the printout of program BROWNIE (see Example 4.1g). These are obtained as in Chapter 3 using the estimators defined below for all the nonzero large sample covariances. The estimation formulae are presented here mainly for reference purposes, and numerical illustrations are omitted.

$$\mathrm{cov}(\hat{f}_i, \hat{S}_i) = \hat{f}_i \hat{S}_i \left[\frac{1}{R_{i\cdot} - R_{ii}} - \frac{1}{N_i}\right], \quad i = \begin{cases} 2,\ldots,k-2 \text{ if } \ell = k \\ 2,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}_{i+1}, \hat{S}_i) = -\hat{f}_{i+1} \hat{S}_i \left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right], \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}_{i+1}, \hat{S}'_i) = -\hat{f}_{i+1} \hat{S}'_i \left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right], \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}_i, \hat{f}''''_i) = -\hat{f}_i \hat{f}''''_i / N_i, \quad i = \begin{cases} 2,\ldots,k-1 \text{ if } \ell = k \\ 2,\ldots,k \quad \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}_i, \hat{S}_{i+1}) = -\hat{S}_i \hat{S}_{i+1} \left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right], \quad i = \begin{cases} 1,\ldots,k-3 \text{ if } \ell = k \\ 1,\ldots,k-2 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}_i, \hat{S}'_i) = \hat{S}_i \hat{S}'_i \left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}} + \frac{1}{D_{i+1} - A_{i+1} - R_{i+1\cdot} + R_{i+1,i+1}} - \frac{1}{D_{i+1} - R_{i+1\cdot}}\right], \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}_{i+1}, \hat{S}'_i) = -\hat{S}_{i+1} \hat{S}'_i \left[\frac{1}{R_{i+1\cdot} - R_{i+1,i+1}} - \frac{1}{N_{i+1}}\right], \quad i = \begin{cases} 1,\ldots,k-3 \text{ if } \ell = k \\ 1,\ldots,k-2 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}'_i, \hat{S}'_i) = -\hat{f}'_i \hat{S}'_i / K_i, \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}'_{i+1}, \hat{S}''_i) = \hat{f}'_{i+1} \hat{S}''_i / K_{i+1}, \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}'_i, \hat{f}'''_i) = -\hat{f}'_i \hat{f}'''_i / K_i, \quad i = \begin{cases} 2,\ldots,k-1 \text{ if } \ell = k \\ 2,\ldots,k \quad \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}'_{i+1}, \hat{S}''_i) = -\hat{S}'_{i+1} \hat{S}''_i \left[\frac{1}{Y_{i+1\cdot} - Y_{i+1,i+1}} - \frac{1}{K_{i+1}}\right], \quad i = \begin{cases} 1,\ldots,k-3 \text{ if } \ell = k \\ 1,\ldots,k-2 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}'_i, \hat{f}'''_i) = \hat{S}'_i \hat{f}'''_i \left[\frac{1}{Y_{i\cdot} - Y_{ii}} - \frac{1}{K_i}\right], \quad i = \begin{cases} 2,\ldots,k-2 \text{ if } \ell = k \\ 2,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}''_i, \hat{S}''_i) = -\hat{f}''_i \hat{S}''_i / M_i, \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}''_i, \hat{f}'''_{i+1}) = -\hat{S}''_i \hat{f}'''_{i+1} \left[\frac{1}{Y_{i+1\cdot} - Y_{i+1,i+1}} - \frac{1}{K_{i+1}}\right], \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}''''_i, \hat{S}_i) = -\hat{f}''''_i \hat{S}_i / N_i, \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}''''_{i+1}, \hat{S}_i) = \hat{f}''''_{i+1} \hat{S}_i / N_{i+1}, \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}''''_{i+1}, \hat{S}'_i) = \hat{f}''''_{i+1} \hat{S}'_i / N_{i+1}, \quad i = \begin{cases} 1,\ldots,k-2 \text{ if } \ell = k \\ 1,\ldots,k-1 \text{ if } \ell > k. \end{cases}$$

Goodness of Fit Test

A goodness of fit test to H_6 is computed by the FORTRAN program in a manner analogous to that used in computing the goodness of fit test to H_3 (Section 3.6). Thus the printout, like that for H_3, consists of a series of contingency tables with resulting chi-square values and degrees of freedom. (The contingency tables are derived from the rows of the recovery matrices as described in Brownie 1973). Individual chi-square values and (separately) the degrees of freedom are summed to give a total chi-square with corresponding degrees of freedom.

In Example 4.1h, we find that for the synthetic data of Table 4.2 the total chi-square value testing goodness of fit for H_6 is 20.16 with 22 df (inadvertently not photographed for Example 4.1h). For real data, such a result would suggest that there is no indication that the assumptions of H_6 are incorrect.

4.4 The Model Under H_5

The model of this section is based on a parameterization which is intermediate in complexity between those of the models under H_4 and H_6. However, it is difficult to find a meaningful biological interpretation for this parameterization, and for this reason the model is considered last, and is included only because the related estimation and testing procedures have been coded in program BROWNIE.

To be consistent we define a hypothesis H_5, with assumptions 1 and 2 the same as those of H_4 and H_6, but with a third assumption that is artificial in that it is dictated by the parameterization and has not been derived from consideration of biological or ecological factors. The model structure, estimation formulae, etc., are presented without accompanying discussion or numerical illustrations.

The assumptions of H_5 are:

(1) Annual survival and recovery rates are year-specific;

(2) annual survival and recovery rates are different for young, subadults, and adults; and

(3) in any year, newly released subadults, and subadults that are survivors of young released the year before, have different recovery rates (namely f_i' and f_i''').

The expected numbers of recoveries under H_5 for $k=3$ and $\ell=5$ are shown in Table 4.5. Note that we cannot attribute the difference in the recovery rates for subadults to the effect of a different reporting rate for new releases, because then it would be logical to assume that the recovery rate for newly released adults is similarly affected, leading to assumption 3 of H_6.

Table 4.5. *Expected numbers of band recoveries under H_5 for a banding study with $k=3$, $\ell=5$, $s=2$.*

Year banded	Number banded	Year of recovery 1	2	3	4	5
		Birds banded and released as adults				
1	N_1	N_1f_1	$N_1S_1f_2$	$N_1S_1S_2f_3$	$N_1S_1S_2S_3f_4$	$N_1S_1S_2S_3S_4f_5$
2	N_2		N_2f_2	$N_2S_2f_3$	$N_2S_2S_3f_4$	$N_2S_2S_3S_4f_5$
3	N_3			N_3f_3	$N_3S_3f_4$	$N_3S_3S_4f_5$
		Birds banded and released as subadults				
1	K_1	K_1f_1'	$K_1S_1'f_2$	$K_1S_1'S_2f_3$	$K_1S_1'S_2S_3f_4$	$K_1S_1'S_2S_3S_4f_5$
2	K_2		K_2f_2'	$K_2S_2'f_3$	$K_2S_2'S_3f_4$	$K_2S_2'S_3S_4f_5$
3	K_3			K_3f_3'	$K_3S_3'f_4$	$K_3S_3'S_4f_5$
		Birds banded and released as young				
1	M_1	M_1f_1''	$M_1S_1''f_2'''$	$M_1S_1''S_2'f_3$	$M_1S_1''S_2'S_3f_4$	$M_1S_1''S_2'S_3S_4f_5$
2	M_2		M_2f_2''	$M_2S_2''f_3'''$	$M_2S_2''S_3'f_4$	$M_2S_2''S_3'S_4f_5$
3	M_3			M_3f_3''	$M_3S_3''f_4'''$	$M_3S_3''S_4'f_5$

Estimation of Parameters

ML estimators of individually estimable annual recovery and survival rates are:

$$\hat{f}_i = \frac{R_{i\cdot}}{N_i}\frac{A_i}{D_i} \qquad , i=1,\ldots,k,$$

$$\hat{f}_i' = \frac{Y_{ii}}{K_i} \qquad , i=1,\ldots,k,$$

$$\hat{f}_i'' = \frac{Q_{ii}}{M_i} \qquad , i=1,\ldots,k,$$

$$\hat{f}_i'' = \frac{Q_{i-1,i}}{Q_{i-1\cdot} - Q_{i-1,i-1} - Q_{i-1,i}} \frac{Y_{i\cdot} - Y_{ii}}{K_i} \quad , i = \begin{cases} 2,\ldots,k-1 & \text{if } \ell = k \\ 2,\ldots,k & \text{if } \ell > k, \end{cases}$$

$$\hat{S}_i = \frac{R_{i\cdot}}{N_i} \frac{D_i - A_i}{D_i} \frac{N_{i+1}}{R_{i+1\cdot}} \quad , i = 1,\ldots,k-1,$$

$$\hat{S}_i' = \frac{Y_{i\cdot} - Y_{ii}}{K_i} \frac{N_{i+1}}{R_{i+1\cdot}} \quad , i = 1,\ldots,k-1,$$

$$\hat{S}_i'' = \frac{Q_{i\cdot} - Q_{ii} - Q_{i,i+1}}{M_i} \frac{K_{i+1}}{Y_{i+1\cdot} - Y_{i+1,i+1}} \quad , i = \begin{cases} 1,\ldots,k-2 & \text{if } \ell = k \\ 1,\ldots,k-1 & \text{if } \ell > k. \end{cases}$$

Estimates obtained by evaluating the above formulae for the data of Table 4.2 are shown in Example 4.1d.

Sampling Variances, Standard Errors, and Confidence Intervals

Confidence intervals for parameters are obtained using the above ML estimators and the estimators of their sampling variances defined below,

$$\text{var}(\hat{f}_i) = (\hat{f}_i)^2 \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} + \frac{1}{A_i} - \frac{1}{D_i}\right] \quad , i = 1,\ldots,k,$$

$$\text{var}(\hat{S}_i) = (\hat{S}_i)^2 \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} + \frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}} + \frac{1}{D_i - A_i} - \frac{1}{D_i}\right] \quad , i = 1,\ldots,k-1,$$

$$\text{var}(\hat{f}_i') = \hat{f}_i'(1 - \hat{f}_i') / K_i \quad , i = 1,\ldots,k,$$

$$\text{var}(\hat{S}_i') = (\hat{S}_i')^2 \left[\frac{1}{Y_{i\cdot} - Y_{ii}} - \frac{1}{K_i} + \frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \quad , i = 1,\ldots,k-1,$$

$$\text{var}(\hat{f}_i'') = \hat{f}_i''(1 - \hat{f}_i'') / M_i \quad , i = 1,\ldots,k,$$

$$\text{var}(\hat{S}_i'') = (\hat{S}_i'')^2 \left[\frac{1}{Q_{i\cdot} - Q_{ii} - Q_{i,i+1}} - \frac{1}{M_i} + \frac{1}{Y_{i+1\cdot} - Y_{i+1,i+1}} - \frac{1}{K_{i+1}}\right] \quad , i = \begin{cases} 1,\ldots,k-2 & \text{if } \ell = k \\ 1,\ldots,k-1 & \text{if } \ell > k, \end{cases}$$

$$\text{var}(\hat{f}_i''') = (\hat{f}_i''')^2 \left[\frac{1}{Y_{i\cdot} - Y_{ii}} - \frac{1}{K_i} + \frac{1}{Q_{i-1\cdot} - Q_{i-1,i-1} - Q_{i-1,i}} + \frac{1}{Q_{i-1,i}}\right] \quad , i = \begin{cases} 2,\ldots,k-1 & \text{if } \ell = k \\ 2,\ldots,k & \text{if } \ell > k. \end{cases}$$

Sampling Covariances and Correlations

Estimators of the nonzero, large-sample covariances between the ML estimators are defined below. Estimates of the corresponding correlations are obtained in the usual way. Thus,

$$\text{cov}(\hat{f}_i, \hat{S}_i) = \hat{f}_i \hat{S}_i \left[\frac{1}{R_{i\cdot}} - \frac{1}{N_i} - \frac{1}{D_i}\right] \quad , i = 1,\ldots,k-1,$$

$$\text{cov}(\hat{f}_{i+1}, \hat{S}_i) = -\hat{f}_{i+1} \hat{S}_i \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \quad , i = 1,\ldots,k-1,$$

$$\text{cov}(\hat{f}_{i+1}, \hat{S}_i') = \hat{f}_{i+1} \hat{S}_i' \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \quad , i = 1,\ldots,k-1,$$

$$\text{cov}(\hat{S}_i, \hat{S}_{i+1}) = -\hat{S}_i \hat{S}_{i+1} \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \quad , i = 1,\ldots,k-2,$$

$$\text{cov}(\hat{S}_{i+1}, \hat{S}_i') = -\hat{S}_{i+1} \hat{S}_i' \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \quad , i = 1,\ldots,k-2,$$

$$\text{cov}(\hat{S}_i, \hat{S}_i') = \hat{S}_i \hat{S}_i' \left[\frac{1}{R_{i+1\cdot}} - \frac{1}{N_{i+1}}\right] \quad , i = 1,\ldots,k-1,$$

$$\mathrm{cov}(\hat{f}_i', \hat{S}_i') = -\hat{f}_i'\hat{S}_i' / K_i \qquad , i = 1, \ldots, k-1,$$

$$\mathrm{cov}(\hat{f}_{i+1}', \hat{S}_i'') = \hat{f}_{i+1}'\hat{S}_i'' / K_{i+1} \qquad , i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{f}_i', \hat{f}_i'') = -\hat{f}_i' \hat{f}_i'' / K_i \qquad , i = \begin{cases} 2, \ldots, k-1 \text{ if } \ell = k \\ 2, \ldots, k \quad \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}_{i+1}', \hat{S}_i'') = -\hat{S}_{i+1}'\hat{S}_i'' \left[\frac{1}{Y_{i+1\cdot} - Y_{i+1,i+1}} - \frac{1}{K_{i+1}}\right] \qquad , i = 1, \ldots, k-2,$$

$$\mathrm{cov}(\hat{S}_i', \hat{f}_i'') = \hat{S}_i' \hat{f}_i'' \left[\frac{1}{Y_{i\cdot} - Y_{ii}} - \frac{1}{K_i}\right] \qquad , i = 2, \ldots, k-1,$$

$$\mathrm{cov}(\hat{f}_i'', \hat{S}_i'') = -\hat{f}_i'' S_i'' / M_i \qquad , i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k, \end{cases}$$

$$\mathrm{cov}(\hat{S}_i'', \hat{f}_{i+1}'') = -\hat{S}_i'' \hat{f}_{i+1}'' \left[\frac{1}{Y_{i+1\cdot} - Y_{i+1,i+1}} - \frac{1}{K_{i+1}}\right] \qquad , i = \begin{cases} 1, \ldots, k-2 \text{ if } \ell = k \\ 1, \ldots, k-1 \text{ if } \ell > k. \end{cases}$$

Estimates of covariances and corresponding correlations obtained by evaluating the above formulae for the data of Table 4.2 are contained in Example 4.1e.

Goodness of Fit Test

A goodness of fit test to the model under H_5 is computed by the FORTRAN program in a manner similar to that used for the goodness of fit test to H_4. For Example 4.1h the chi-square value testing goodness of fit for H_5 is 22.11 with 26 df (inadvertently not photographed for Example 4.1h). Such a value is consistent with the assumptions of H_5.

4.5 Testing Between Models

As described in Chapters 2 and 3, it is important to choose the model which seems most appropriate for a given data set, in the sense that the model has sufficient parameters to provide an adequate description of the data, but not so many as to make estimation inefficient. This is accomplished by examining the results of goodness of fit tests to each model and the results of tests which compare one model with another. The latter tests are the subject of this section. As before, we always compare a simpler model with a more general alternative, and if the resulting chi-square value is significantly large, the simple model is rejected in favor of the more general one.

Because of the artificial nature of the H_5 assumptions, the important test is comparing H_4 with H_6. The corresponding test statistic is not printed out by the FORTRAN program, but is easily obtained from the test statistics for the tests of H_4 against H_5 and H_5 against H_6, as described below.

H_4 vs. H_5

The test statistic for comparing H_4 with H_5 is obtained as the sum of the single degrees of freedom chi-square statistics from each of the contingency tables

Y_{ii}	$Y_{i\cdot} - Y_{ii}$	$Y_{i\cdot}$
$Q_{i-1,i}$	$Q_{i-1\cdot} - Q_{i-1,i-1} - Q_{i-1,i}$	$Q_{i-1\cdot} - Q_{i-1,i-1}$
		$Y_{i\cdot} + Q_{i-1\cdot} - Q_{i-1,i-1}$

$$, i = \begin{cases} 2, \ldots, k-1 \text{ if } \ell = k \\ 2, \ldots, k \quad \text{ if } \ell > k, \end{cases}$$

The format of the printout of program BROWNIE is like that for the tests between the models of Chapter 3, as can be seen in Example 4.1h. Thus, individual contingency tables and chi-square values are printed, as well as the "TOTAL CHI-SQUARE" and corresponding degrees of freedom (which are $k-2$ if $\ell = k$ and $k-1$ if $\ell > k$).

As discussed before (see Section 3.7), information can be obtained by examining individual chi-square values as well as the total chi-square value. Each contingency table provides a test of the equality of f_i' and f_i'''. Significantly large chi-square values are taken as evidence that inequality exists.

For the synthetic data of Table 4.2, which was generated using the model under H_4 as the probability model, the test results in a nonsignificant chi-square value of 4.322 with $k-2=4$ df (see Example 4.1h).

H_5 *vs.* H_6

This test is based on the contingency tables

R_{ii}	$R_{i\cdot} - R_{ii}$	$R_{i\cdot}$
$A_i - R_{ii}$	$D_i - R_{i\cdot} - A_i + R_{ii}$	$D_i - R_{i\cdot}$
		D_i

$$, i = \begin{cases} 2, \ldots, k-1 & \text{if } \ell = k \\ 2, \ldots, k & \text{if } \ell > k, \end{cases}$$

and is otherwise analogous to the test described above. Similarly, the printout is like that for the test above (see Example 4.1h). The degrees of freedom for the total chi-square value are $k-2$ if $\ell = k$ and $k-1$ if $\ell > k$.

Each contingency table provides a test of the equality of f_i'''' and f_i. Large chi-square values are taken as indicating that inequality exists, and that the H_6 assumptions are more appropriate than those of H_5.

For the synthetic data of Table 4.2, the total chi-square value is 1.947 with 4 df (Example 4.1h).

H_4 *vs* H_6

The test of H_4 against the alternative H_6 tests the validity of assumption 3 of H_4 against the alternative that new releases have a different reporting rate from that for survivors of earlier releases (assumption 3 of H_6). The chi-square test statistic is obtained as the sum of the test statistics for the two tests described above. The degrees of freedom, obtained by summing analogously, are $k-2+k-2=2k-4$ if $\ell = k$ and $k-1+k-1=2k-2$ if $\ell > k$. For example, using results of Example 4.1h, the chi-square value and degrees of freedom for the test of H_4 vs. H_6 for the data of Table 4.2 are $4.322+1.947=6.269$ and $4+4=8$, respectively. With such a result there is no reason to discard H_4 in favor of H_6.

Choosing between Chapter 4 models and their relationship to Chapter 3 models is discussed further in Section 4.6.

An Example

This example consists of the complete output from the FORTRAN program for analysis of the data of Table 4.2. The output is similar in format to that for the Chapter 3 models, but is not as fully documented. The output is self-explanatory with the exception of the following points.

The recovery matrices are printed out in the order – adults, young, subadults. This is different from the (more natural) way the data have been presented in Table 4.2.

The basic subtotals printed are also in a different order from those in Table 4.2. The subtotals labeled W(I), Z(I) and B(I) can be ignored. Those labeled SUMAB(I) are the same as those labeled D_i in Table 4.2 except for the k^{th} one.

An important point to note is that the program will not accept data sets with $\ell = k+1$ (i.e., $s=1$). An error message is printed if such a data set is read in, and no computations are performed.

As usual, estimates labeled F(I), S(I), etc., represent $\hat{f}_i$, $\hat{S}_i$, etc. Similarly COVAR(F(I), S(I)) represent $\text{cov}(\hat{f}_i, \hat{S}_i)$.

Estimates under H_6 labeled SK − 1 FK, S′K − 1 FK, S″K − 1 F‴K can be ignored. These are estimates of products of parameters which are of little interest.

Estimates of survival of over 100% and large confidence intervals have already been noted. Clearly, with adult recovery rates of approximately 5%, releases each year of 700-1,000 birds in each age class are not sufficient to provide reliable estimates of the different survival rates.

The results of the goodness of fit tests, as noted above, indicate that the simplest model (i.e., the model under H_4) seems adequate, and this is borne out by the tests between models. This is not surprising because, as stated earlier, the data were generated stochastically using H_4 as the probability model, with values of $f_i, S_i, f_i', S_i'; f_i'', S_i''$ which are not improbable. Comparison of the estimates (particularly the H_4 estimates) with the actual parameter values used in generating the data may be of interest; Table 4.6 gives the true parameters used in generating the data of Table 4.2. However, we emphasize that this data set represents a single sample and is not a simulation study on which we can base conclusions concerning precision and bias of estimators, power of tests, and so on.

Table 4.6. *Parameters used in generating the synthetic data used for illustrative purposes in Chapter 4.*

i	f_i	S_i	f_i'	S_i'	f_i''	S_i''
1	0.05	0.70	0.10	0.55	0.15	0.40
2	0.04	0.80	0.08	0.65	0.12	0.50
3	0.06	0.83	0.12	0.68	0.18	0.53
4	0.05	0.77	0.10	0.62	0.15	0.47
5	0.07	0.86	0.13	0.71	0.19	0.56
6	0.03	—	0.07	—	0.11	—
Averages	0.05	0.79	0.10	0.64	0.15	0.49

4.6 Summary

The methods of this chapter represent an extension of the methods of Chapters 2 and 3 to the more complex situation where young, subadults, and adults are thought to have different survival and recovery rates, and data are recorded separately for these three age groups. Similarly, analogous procedures can be developed for even more complex situations where data are recorded separately for four or more age classes and survival and recovery rates are specific to each age class. However, the limited applicability of the methods in Chapter 4 suggests that further extensions are of little value at this time, and the methods of Chapters 2 and 3 seem adequate for the analysis of most bird banding data we have encountered.

We have seen that as the complexity (or number of parameters) of the models increases, the number of birds which must be banded to obtain reliable estimates of survival also increases. This factor precludes application of the Chapter 4 methods to other larger game animals (e.g., deer) where age may be more easily determined but the tagging of thousands of animals is not feasible. Thus, in Example 4.1, where H_4 is correct, and adult recovery rates are approximately 5%, then banding 700 or more of each age class, i.e., over 2,000 birds annually, is not sufficient to yield confidence intervals of a reasonable length. If survival and recovery rates are appreciably different for subadults, then using a simpler model such as H_1 or H_2 will result in biased estimators (but shorter confidence intervals), and as usual there is a tradeoff between loss in accuracy and gain in precision. The H_1 estimates for the synthetic data of Table 4.2 are shown in Example 4.2 to illustrate this point. Comparison of the H_1 confidence intervals for adult survival rates with the true parameter values in Table 4.6 shows that only 1 of the 5 confidence intervals includes the true value.

Finally, if analysis of a given data set by Chapter 4 methods (three age classes) indicates that H_4 is preferable to H_6, it is possible that H_4 is too general and that a two-age-class model of Chapter 3 may be adequate. The data for adults and subadults should then be combined and a Chapter 3 analysis performed. (Alternatively, a Chapter 3 analysis could be carried out first and the Chapter 4 analysis used only if it is indicated). If H_1 and H_2 are rejected in favor of H_3, then the Chapter 4 methods are appropriate. However, note that H_3 and H_4 are not strictly comparable in the sense that one is more general than the other, because the H_3 parameterization allows the assumption that reporting rates are different for new releases whereas that of H_4 does not. Nevertheless, rejection of H_1 and H_2 in favor of H_3 is one indication that subadults have different survival and recovery rates and that a Chapter 4 model may be appropriate.

This is illustrated in Example 4.2 which consists of portions of a printout obtained by combining the data for adults and subadults in Example 4.1, and analyzing the combined data and the data for young using the two-age-class models of Chapter 3. The combined recovery data for adults and subadults are printed as the "INPUT MATRIX" for adults. Note that the chi-square values for H_1 vs. H_2 and for H_2 vs. H_3 are both significantly large and H_1 and H_2 are thus rejected in favor of H_3. Also, the H_3 model is the only one for which the goodness of fit test is not significant ($\chi^2 = 11.28$, df = 12). This confirms that, as indicated in Example 4.1, the H_4 model is appropriate for these data.

Example 4.1a

SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

ADULTS INPUT MATRIX

1	700.	36.	20.	26.	11.	17.	6.
2	700.	0.	23.	31.	22.	25.	16.
3	700.	0.	0.	42.	24.	23.	11.
4	700.	0.	0.	0.	38.	37.	20.
5	700.	0.	0.	0.	0.	38.	19.
6	700.	0.	0.	0.	0.	0.	23.

YOUNG INPUT MATRIX

1	1000.	161.	34.	21.	7.	18.	4.
2	1000.	0.	109.	73.	12.	16.	7.
3	1000.	0.	0.	171.	51.	26.	13.
4	1000.	0.	0.	0.	157.	57.	10.
5	1000.	0.	0.	0.	0.	198.	39.
6	1000.	0.	0.	0.	0.	0.	102.

SUBADULTS INPUT MATRIX

1	800.	80.	12.	20.	12.	19.	3.
2	800.	0.	57.	33.	17.	34.	7.
3	800.	0.	0.	83.	26.	25.	13.
4	800.	0.	0.	0.	64.	25.	14.
5	800.	0.	0.	0.	0.	115.	26.
6	800.	0.	0.	0.	0.	0.	57.

BASIC SUBTOTALS

I	RROW(I)	RCOL(I)	QROW(I)	QCOL(I)	T(I)	U(I)	W(I)	Z(I)	YROW(I)	YCOL(I)	V(I)	A(I)	B(I)	SUMAB(I)
1	116.00	36.00	245.00	161.00	116.00	245.00	36.00	0.0	146.00	80.00	146.00	36.00	0.0	116.00
2	117.00	43.00	217.00	143.00	197.00	301.00	77.00	80.00	148.00	69.00	214.00	55.00	80.00	263.00
3	100.00	99.00	261.00	265.00	254.00	419.00	193.00	204.00	147.00	136.00	292.00	173.00	208.00	449.00
4	95.00	95.00	224.00	227.00	250.00	378.00	165.00	219.00	103.00	119.00	259.00	169.00	276.00	470.00
5	57.00	140.00	237.00	315.00	212.00	388.00	257.00	239.00	141.00	218.00	281.00	303.00	301.00	436.00
6	23.00	95.00	102.00	175.00	95.00	175.00	168.00	106.00	57.00	120.00	120.00	192.00	133.00	0.0

Example 4.1b

SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

ESTIMATES UNDER H4

		F(I)			S(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.0514	0.0083	0.0351 - 0.0678	0.6838	0.0922	0.5030 - 0.8645
2	2	0.0350	0.0051	0.0249 - 0.0450	0.9253	0.1196	0.6910 - 1.1597
3	3	0.0550	0.0061	0.0432 - 0.0669	0.6471	0.0893	0.4719 - 0.8222
4	4	0.0488	0.0055	0.0379 - 0.0597	1.0674	0.1735	0.7274 - 1.4073
5	5	0.0566	0.0074	0.0421 - 0.0711	0.7560	0.1903	0.3829 - 1.1290
6	6	0.0329	0.0067	0.0197 - 0.0461			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$		= 0.0466	0.0027	0.0413 - 0.0519	$\bar{S}$ = 0.8159	0.0396	0.7382 - 0.8936

		F'(I)			S'(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.1000	0.0106	0.0792 - 0.1208	0.4936	0.0716	0.3533 - 0.6339
2	2	0.0726	0.0080	0.0569 - 0.0883	0.7870	0.1022	0.5867 - 0.9873
3	3	0.1124	0.0101	0.0927 - 0.1322	0.5256	0.0759	0.3770 - 0.6743
4	4	0.0767	0.0084	0.0603 - 0.0932	0.6390	0.1147	0.4142 - 0.8638
5	5	0.1457	0.0121	0.1221 - 0.1694	0.9284	0.2471	0.4440 - 1.4128
6	6	0.0712	0.0091	0.0534 - 0.0891			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$		= 0.0964	0.0040	0.0886 - 0.1043	$\bar{S}'$ = 0.6747	0.0618	0.5536 - 0.7959

		F''(I)			S''(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.1610	0.0116	0.1382 - 0.1838	0.4541	0.0582	0.3400 - 0.5681
2	2	0.1090	0.0099	0.0897 - 0.1283	0.5878	0.0691	0.4524 - 0.7231
3	3	0.1710	0.0119	0.1477 - 0.1943	0.6990	0.0953	0.5123 - 0.8857
4	4	0.1570	0.0115	0.1345 - 0.1795	0.3801	0.0534	0.2754 - 0.4849
5	5	0.1980	0.0126	0.1733 - 0.2227	0.5474	0.1107	0.3303 - 0.7644
6	6	0.1020	0.0096	0.0832 - 0.1208			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}''$		= 0.1497	0.0046	0.1407 - 0.1587	$\bar{S}''$ = 0.5337	0.0360	0.4632 - 0.6042

Example 4.1c

SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H4

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(F(I+1),S(I))	CORR(F(I+1),S(I))
1	1	-0.000050236	-0.065253606	-0.000170131	-0.359974113
2	2	0.000107256	0.175040771	-0.000436564	-0.602452058
3	3	0.000225954	0.417278723	-0.000287267	-0.580407447
4	4	0.000363052	0.377827136	-0.000973392	-0.757871878
5	5	0.000591297	0.419539670	-0.001044490	-0.814461397

I	YR	COVAR(F(I+1),S'(I))	CORR(F(I+1),S'(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1	-0.000122813	-0.334865286	-0.004503824	-0.408482207
2	2	-0.000371326	-0.599501246	-0.005131979	-0.480436117
3	3	-0.000233370	-0.555277654	-0.006283324	-0.405460569
4	4	-0.000582744	-0.686188617	-0.013003692	-0.393878819
5	5	-0.001282716	-0.770326529		

I	YR	COVAR(S(I),S'(I))	CORR(S(I),S'(I))	COVAR(S(I+1),S'(I))	CORR(S(I+1),S'(I))
1	1	0.002402449	0.364044173	-0.003251200	-0.379989949
2	2	0.006242327	0.510889461	-0.004365079	-0.478082769
3	3	0.003094347	0.456530305	-0.005104426	-0.387904958
4	4	0.010991640	0.552531049	-0.007784970	-0.356623619
5	5	0.029512953	0.627401268		

I	YR	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))	COVAR(F'(I+1),S''(I))	CORR(F'(I+1),S''(I))
1	1	-0.000061699	-0.081286439	-0.000181438	-0.389404703
2	2	0.000068329	0.083470093	-0.000366871	-0.526880039
3	3	0.000096383	0.126032913	-0.000453615	-0.567282106
4	4	0.000160665	0.166881758	-0.000323680	-0.502114632
5	5	0.000139979	0.046960028	-0.000635460	-0.630907169

I	YR	COVAR(S'(I+1),S''(I)),	CORR(S'(I+1),S''(I)),	COVAR(F''(I),S''(I))	CORR(F''(I),S''(I))
1	1	-0.001967903	-0.331048945	-0.000073103	-0.108132806
2	2	-0.001715530	-0.327366480	-0.000064065	-0.094112224
3	3	-0.003778400	-0.345847340	-0.000119534	-0.105394170
4	4	-0.002061862	-0.156096394	-0.000059682	-0.097064273
5	5			-0.000108379	-0.077660210

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

Example 4.1d

SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

ESTIMATES UNDER H5

		F(I)			S(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.0514	0.0083	0.0351 - 0.0678	0.6838	0.0922	0.5030 - 0.8645
2	2	0.0350	0.0051	0.0249 - 0.0450	0.9253	0.1196	0.6910 - 1.1597
3	3	0.0550	0.0061	0.0432 - 0.0669	0.6471	0.0893	0.4719 - 0.8222
4	4	0.0488	0.0055	0.0379 - 0.0597	1.0674	0.1735	0.7274 - 1.4073
5	5	0.0566	0.0074	0.0421 - 0.0711	0.7560	0.1903	0.3829 - 1.1290
6	6	0.0329	0.0067	0.0197 - 0.0461			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$		= 0.0466	0.0027	0.0413 - 0.0519	$\bar{S}$ = 0.8159	0.0396	0.7382 - 0.8936

		F'(I)			S'(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.1000	0.0106	0.0792 - 0.1208	0.4936	0.0716	0.3533 - 0.6339
2	2	0.0712	0.0091	0.0534 - 0.0891	0.7962	0.1077	0.5851 - 1.0074
3	3	0.1037	0.0108	0.0826 - 0.1249	0.5895	0.0903	0.4125 - 0.7665
4	4	0.0800	0.0096	0.0612 - 0.0988	0.5987	0.1205	0.3625 - 0.8348
5	5	0.1437	0.0124	0.1194 - 0.1681	0.9891	0.2785	0.4433 - 1.5349
6	6	0.0712	0.0091	0.0534 - 0.0891			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$		= 0.0950	0.0042	0.0867 - 0.1033	$\bar{S}'$ = 0.6934	0.0684	0.5594 - 0.8275

		F''(I)			S''(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.1610	0.0116	0.1382 - 0.1838	0.4396	0.0745	0.2935 - 0.5856
2	2	0.1090	0.0099	0.0897 - 0.1283	0.4375	0.0896	0.2619 - 0.6131
3	3	0.1710	0.0119	0.1477 - 0.1943	0.8000	0.1771	0.4528 - 1.1472
4	4	0.1570	0.0115	0.1345 - 0.1795	0.3077	0.1136	0.0851 - 0.5303
5	5	0.1980	0.0126	0.1733 - 0.2227			
6	6	0.1020	0.0096	0.0832 - 0.1208			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}''$		= 0.1497	0.0046	0.1407 - 0.1587	$\bar{S}''$ = 0.4962	0.0601	0.3783 - 0.6141

		F'''(I)			S''K-1 F'''K		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1				0.0390	0.0061	0.0270 - 0.0510
2	2	0.0773	0.0188	0.0405 - 0.1142			
3	3	0.1669	0.0397	0.0890 - 0.2447			
4	4	0.0637	0.0168	0.0308 - 0.0967			
5	5	0.1853	0.0729	0.0424 - 0.3281			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL			
$\bar{F}'''$		= 0.1233	0.0217	0.0808 - 0.1658			

Example 4.1e

SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H5

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(F(I+1),S(I))	CORR(F(I+1),S(I))
1	1	-0.000050236	-0.065253596	-0.000170131	-0.359974129
2	2	0.000107256	0.175040771	-0.000436564	-0.602452058
3	3	0.000225955	0.417278884	-0.000287268	-0.580407783
4	4	0.000363052	0.377827136	-0.000973392	-0.757871878
5	5	0.000591298	0.419539808	-0.001044490	-0.814461310

I	YR	COVAR(F(I+1),S'(I))	CORR(F(I+1),S'(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1	-0.000122813	-0.334865325	-0.004503824	-0.408482036
2	2	-0.000375668	-0.575267452	-0.005131982	-0.480436343
3	3	-0.000261705	-0.523084352	-0.006283324	-0.405460471
4	4	-0.000545969	-0.611926022	-0.013003699	-0.393878834
5	5	-0.001366614	-0.728368381		

I	YR	COVAR(S(I+1),S'(I))	CORR(S(I+1),S'(I))	COVAR(S(I),S'(I))	CORR(S(I),S'(I))
1	1	-0.003251200	-0.379989949	0.002402450	0.364044263
2	2	-0.004416123	-0.458757424	0.006315321	0.490237920
3	3	-0.005724199	-0.365415640	0.003470059	0.430062293
4	4	-0.007293694	-0.318028380	0.010297995	0.492733690
5	5			0.031443290	0.593228061

I	YR	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))	COVAR(F'(I+1),S''(I))	CORR(F'(I+1),S''(I))
1	1	-0.000061699	-0.081286439	0.000039148	0.057764651
2	2	-0.000070916	-0.072368354	0.000056738	0.058733650
3	3	-0.000076447	-0.078514990	0.000080000	0.047083209
4	4	-0.000059868	-0.051801824	0.000055288	0.039251102
5	5	-0.000177734	-0.051455091		

I	YR	COVAR(F'(I),F'''(I))	CORR(F'(I),F'''(I))	COVAR(S'(I+1),S''(I))	CORR(S'(I+1),S''(I))
1	1			-0.003408651	-0.424550718
2	2	-0.000006889	-0.040264081	-0.003707234	-0.458119236
3	3	-0.000021639	-0.050542093	-0.011682007	-0.547304148
4	4	-0.000006375	-0.039507277	-0.011325236	-0.358129930
5	5	-0.000033287	-0.036824414		

I	YR	COVAR(S'(I),F'''(I))	CORR(S'(I),F'''(I))	COVAR(F''(I),S''(I))	CORR(F''(I),S''(I))
1	1			-0.000070769	-0.081713908
2	2	0.000599824	0.295927492	-0.000047687	-0.054004120
3	3	0.001413892	0.394225527	-0.000136800	-0.064860548
4	4	0.000930910	0.459240230	-0.000048308	-0.036976801
5	5	0.006818503	0.335988627		

I	YR	COVAR(S''(I),F'''(I+1)	CORR(S''(I),F'''(I+1)
1	1	-0.000331126	-0.236210267
2	2	-0.001049373	-0.294903029
3	3	-0.001243941	-0.417408350
4	4	-0.002121057	-0.256299700

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

Example 4.1f

SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

ESTIMATES UNDER H6

		F(I)			S(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1				0.6645	0.0990	0.4704 - 0.8586
2	2	0.0377	0.0084	0.0213 - 0.0541	1.0123	0.1656	0.6879 - 1.3368
3	3	0.0498	0.0083	0.0335 - 0.0661	0.6621	0.1209	0.4251 - 0.8991
4	4	0.0437	0.0073	0.0294 - 0.0580	0.9024	0.2446	0.4230 - 1.3817
5	5	0.0631	0.0159	0.0319 - 0.0943			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$		= 0.0486	0.0053	0.0382 - 0.0589	$\bar{S}$ = 0.8103	0.0606	0.6915 - 0.9292

		F'(I)			S'(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.1000	0.0106	0.0792 - 0.1208	0.4797	0.0759	0.3309 - 0.6285
2	2	0.0712	0.0091	0.0534 - 0.0891	0.8575	0.1416	0.5800 - 1.1351
3	3	0.1037	0.0108	0.0826 - 0.1249	0.6393	0.1142	0.4154 - 0.8631
4	4	0.0800	0.0096	0.0612 - 0.0988	0.5402	0.1544	0.2375 - 0.8429
5	5	0.1437	0.0124	0.1194 - 0.1681			
6	6	0.0712	0.0091	0.0534 - 0.0891			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$		= 0.0950	0.0042	0.0867 - 0.1033	$\bar{S}'$ = 0.6292	0.0626	0.5065 - 0.7519

		F''(I)			S''(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.1610	0.0116	0.1382 - 0.1838	0.4396	0.0745	0.2935 - 0.5856
2	2	0.1090	0.0099	0.0897 - 0.1283	0.4375	0.0896	0.2619 - 0.6131
3	3	0.1710	0.0119	0.1477 - 0.1943	0.8000	0.1771	0.4528 - 1.1472
4	4	0.1570	0.0115	0.1345 - 0.1795	0.3077	0.1136	0.0851 - 0.5303
5	5	0.1980	0.0126	0.1733 - 0.2227			
6	6	0.1020	0.0096	0.0832 - 0.1208			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}''$		= 0.1497	0.0046	0.1407 - 0.1587	$\bar{S}''$ = 0.4962	0.0601	0.3783 - 0.6141

		F'''(I)			F''''(I)		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1				0.0514	0.0083	0.0351 - 0.0678
2	2	0.0773	0.0188	0.0405 - 0.1142	0.0329	0.0067	0.0197 - 0.0461
3	3	0.1669	0.0397	0.0890 - 0.2447	0.0600	0.0090	0.0424 - 0.0776
4	4	0.0637	0.0168	0.0308 - 0.0967	0.0543	0.0086	0.0375 - 0.0711
5	5	0.1852	0.0729	0.0424 - 0.3281	0.0543	0.0086	0.0375 - 0.0711
6	6				0.0329	0.0067	0.0197 - 0.0461
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'''$		= 0.1233	0.0217	0.0808 - 0.1658	$\bar{F}''''$ = 0.0476	0.0033	0.0412 - 0.0541

		SK-1 FK			S'K-1 FK		
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	****	0.0271	0.0061	0.0151 - 0.0392	0.0325	0.0063	0.0202 - 0.0448

		S''K-1 F'''K					
I	YR	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	****	0.0390	0.0061	0.0270 - 0.0510			

Example 4.1g

SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H6

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(F(I+1),S(I))	CORR(F(I+1),S(I))
1	1			-0.000230693	-0.278555120
2	2	0.000351440	0.253809794	-0.000797047	-0.577502687
3	3	0.000521272	0.517080767	-0.000466452	-0.528731567
4	4	0.000635744	0.356325046	-0.002915267	-0.748273085

I	YR	COVAR(F(I+1),S'(I))	CORR(F(I+1),S'(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1	-0.000166532	-0.262337648	-0.006195705	-0.377935379
2	2	-0.000675158	-0.571871189	-0.010598645	-0.529396925
3	3	-0.000450368	-0.540507844	-0.009627990	-0.325556066
4	4	-0.001745325	-0.709376989		

I	YR	COVAR(S(I),S'(I))	CORR(S(I),S'(I))	COVAR(S(I+1),S'(I))	CORR(S(I+1),S'(I))
1	1	0.003548755	0.472181863	-0.004472524	-0.355931937
2	2	0.015222251	0.649245835	-0.008977845	-0.524234652
3	3	0.007426511	0.537706946	-0.009295996	-0.332807022
4	4	0.027951255	0.740024763		

I	YR	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))	COVAR(F'(I+1),S''(I))	CORR(F'(I+1),S''(I))
1	1	-0.000059963	-0.074485214	0.000039148	0.057764651
2	2	-0.000076374	-0.059296152	0.000056738	0.058733650
3	3	-0.000082903	-0.067327013	0.000080000	0.047083209
4	4	-0.000054024	-0.036468897	0.000055288	0.039251102

I	YR	COVAR(F'(I),F'''(I))	CORR(F'(I),F'''(I))	COVAR(S'(I+1),S''(I))	CORR(S'(I+1),S''(I))
1	1			-0.003671008	-0.347862263
2	2	-0.000006889	-0.040264081	-0.004020289	-0.392839777
3	3	-0.000021639	-0.050542093	-0.010541543	-0.385306504
4	4	-0.000006375	-0.039507297		
5	5	-0.000033287	-0.036824411		

I	YR	COVAR(S'(I),F'''(I))	CORR(S'(I),F'''(I))	COVAR(F''(I),S''(I))	CORR(F''(I),S''(I))
1	1			-0.000070769	-0.081713908
2	2	0.000645991	0.242472765	-0.000047687	-0.054004120
3	3	0.001533288	0.338050750	-0.000136800	-0.064860548
4	4	0.000840028	0.323308882	-0.000048308	-0.036976801

I	YR	COVAR(S''(I),F'''(I+1))	CORR(S''(I),F'''(I+1))	COVAR(F''''(I),F(I))	CORR(F''''(I),F(I))
1	1	-0.000331126	-0.236210267		
2	2	-0.001049373	-0.294903029	-0.000001769	-0.031397738
3	3	-0.001243940	-0.417408472	-0.000004268	-0.057032674
4	4	-0.002121056	-0.256299725	-0.000003390	-0.054265385
5	5			-0.000004893	-0.035865676

I	YR	COVAR(F''''(I),S(I))	CORR(F''''(I),S(I))	COVAR(F''''(I+1),S(I))	CORR(F''''(I+1),S(I))
1	1	-0.000048823	-0.059061558	0.000031192	0.046752769
2	2	-0.000047518	-0.042599510	0.000086773	0.058391125
3	3	-0.000056750	-0.052281857	0.000051345	0.049579505
4	4	-0.000069980	-0.033412822	0.000069980	0.033412822

I	YR	COVAR(F''''(I+1),S'(I))	CORR(F''''(I+1),S'(I))
1	1	0.000022517	0.044030805
2	2	0.000073503	0.057821740
3	3	0.000049575	0.050683783
4	4	0.000041896	0.031675984

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

Example 4.1h

```
SYNTHETIC DATA GENERATED FOR THE THREE AGE CLASS CASE

CHI-SQUARE TEST OF H4 VS H5
---------------------------

     2 X 2 CONTINGENCY TABLE    CORRESPONDING CHI-SQUARE STATISTIC
                                WITH 1 DEGREE OF FREEDOM
     -----------------------    ----------------------------------

I= 2         57      91
             34      50                        0.087

I= 3         83      64
             73      35                        3.247

I= 4         64      39
             51      39                        0.597

I= 5        115      26
             57      10                        0.392
|-------------------------------------------------------------|
|TOTAL CHI-SQUARE WITH  4 DEGREES OF FREEDOM =    4.322|
|-------------------------------------------------------------|
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN     4.32  = 0.36413


CHI-SQUARE TEST OF H5 VS H6
---------------------------

     2 X 2 CONTINGENCY TABLE    CORRESPONDING CHI-SQUARE STATISTIC
                                WITH 1 DEGREE OF FREEDOM
     -----------------------    ----------------------------------

I= 2         23      94
             32     114                        0.201

I= 3         42      58
            131     218                        0.654

I= 4         38      57
            131     244                        0.845

I= 5         38      19
            265     114                        0.248
|-------------------------------------------------------------|
|TOTAL CHI-SQUARE WITH  4 DEGREES OF FREEDOM =    1.947|
|-------------------------------------------------------------|
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN     1.95  = 0.74550


CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H6
-----------------------------------------------------

               CONTINGENCY TABLES                          CORRESPONDING CHI-SQUARE
                                                           STATISTICS AND DEGREES OF FREEDOM
               ------------------                          ---------------------------------

        I= 1
                 6. 11. 20. 17. 26.
                 3. 12. 12. 19. 20.                         2.62 WITH  4 D.F.

        I= 2
                16. 22. 25. 31.
                 7. 17. 34. 33.
                 4.  7. 18. 21.
                 9. 23. 36. 46.                            10.11 WITH  9 D.F.

        I= 3
                11. 24. 23.
                13. 26. 25.
                 7. 12. 16.
                36. 69.113.                                 5.13 WITH  6 D.F.

        I= 4
                20. 37.
                14. 25.
                13. 26.
                67.177.                                     2.30 WITH  3 D.F.


CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H4
-----------------------------------------------------

                                    |-------------------------------------|
                                    |TOTAL CHI-SQUARE     26.43 WITH 30 D.F.|
                                    |-------------------------------------|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN    26.43  = 0.65322
```

Example 4.2a

SYNTHETIC DATA GENERATED FOR 3 AGE CLASSES WHERE ADULTS & SUBADULTS ARE GROUPED

ADULTS INPUT MATRIX

1	1500.	116.	32.	46.	23.	36.	9.
2	1500.	0.	80.	64.	39.	59.	23.
3	1500.	0.	0.	125.	50.	48.	24.
4	1500.	0.	0.	0.	102.	62.	34.
5	1500.	0.	0.	0.	0.	153.	45.
6	1500.	0.	0.	0.	0.	0.	80.

YOUNG INPUT MATRIX

1	1000.	161.	34.	21.	7.	18.	4.
2	1000.	0.	109.	73.	12.	16.	7.
3	1000.	0.	0.	171.	51.	26.	13.
4	1000.	0.	0.	0.	157.	57.	10.
5	1000.	0.	0.	0.	0.	198.	39.
6	1000.	0.	0.	0.	0.	0.	102.

THE HYPOTHESIS H1. (SEE BROWNIE AND ROBSON, 1974. CORNELL BIOMETRICS UNIT PAPER NO. BU-514-M)

ASSUMPTIONS: (1) ANNUAL SURVIVAL AND RECOVERY RATES ARE YEAR-SPECIFIC.

(2) YOUNG BIRDS HAVE DIFFERENT SURVIVAL AND RECOVERY RATES FROM THOSE OF ADULTS.

PARAMETERS:

F(I) = BAND RECOVERY RATE FOR ADULTS IN YEAR I.
S(I) = SURVIVAL RATE FOR ADULTS IN YEAR I.
F'(I) = BAND RECOVERY RATE FOR YOUNG IN YEAR I.
S'(I) = SURVIVAL RATE FOR YOUNG IN YEAR I.

STRUCTURE OF THE MODEL UNDER H1 (IN TERMS OF EXPECTED NUMBERS OF BAND RETURNS):

BANDED AS ADULTS

N(1)F(1)	N(1)S(1)F(2)	N(1)S(1)S(2)F(3)	N(1)S(1)S(2)S(3)F(4)
	N(2)F(2)	N(2)S(2)F(3)	N(2)S(2)S(3)F(4)
		N(3)F(3)	N(3)S(3)F(4)

BANDED AS YOUNG

M(1)F'(1)	M(1)S'(1)F(2)	M(1)S'(1)S(2)F(3)	M(1)S'(1)S(2)S(3)F(4)
	M(2)F'(2)	M(2)S'(2)F(3)	M(2)S'(2)S(3)F(4
		M(3)F'(3)	M(3)S'(3)F(4)

ESTIMATES UNDER H1

I	YR	F(I) ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	S(I) ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.0773	0.0069	0.0638 - 0.0909	0.5492	0.0529	0.4455 - 0.6530
2	2	0.0521	0.0046	0.0430 - 0.0612	0.7539	0.0646	0.6273 - 0.8804
3	3	0.0770	0.0054	0.0663 - 0.0876	0.6616	0.0628	0.5385 - 0.7847
4	4	0.0565	0.0045	0.0477 - 0.0654	0.5691	0.0566	0.4582 - 0.6801
5	5	0.0974	0.0068	0.0840 - 0.1108	0.6419	0.0921	0.4614 - 0.8224
6	6	0.0533	0.0058	0.0420 - 0.0647			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}$		= 0.0689	0.0024	0.0643 - 0.0736	$\bar{S}$ = 0.6352	0.0202	0.5957 - 0.6746

I	YR	F'(I) ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	S'(I) ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
1	1	0.1610	0.0116	0.1382 - 0.1838	0.4740	0.0561	0.3640 - 0.5840
2	2	0.1090	0.0099	0.0897 - 0.1283	0.6537	0.0705	0.5154 - 0.7919
3	3	0.1710	0.0119	0.1477 - 0.1943	0.6788	0.0817	0.5187 - 0.8390
4	4	0.1570	0.0115	0.1345 - 0.1795	0.5054	0.0684	0.3713 - 0.6394
5	5	0.1980	0.0126	0.1733 - 0.2227	0.7227	0.1380	0.4522 - 0.9932
6	6	0.1020	0.0096	0.0832 - 0.1208			
		AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL	AVERAGE ESTIMATE	STANDARD ERROR	95% CONFIDENCE INTERVAL
$\bar{F}'$		= 0.1497	0.0046	0.1407 - 0.1587	$\bar{S}'$ = 0.6069	0.0393	0.5300 - 0.6839

Example 4.2b

SYNTHETIC DATA GENERATED FOR 3 AGE CLASSES WHERE ADULTS & SUBADULTS ARE GROUPED

ESTIMATED NON-ZERO COVARIANCES AND CORRELATIONS UNDER H1

I	YR	COVAR(F(I),S(I))	CORR(F(I),S(I))	COVAR(S(I),S(I+1))	CORR(S(I),S(I+1))
1	1	-0.000028316	-0.077556398	-0.001286452	-0.376411009
2	2	0.000042690	0.142444283	-0.001686778	-0.415990429
3	3	0.000099862	0.292174826	-0.001650718	-0.464165164
4	4	0.000092539	0.361385704	-0.001601589	-0.307144874
5	5	0.000176929	0.280870589		

I	YR	COVAR(F(I+1),S(I))	CORR(F(I+1),S(I))	COVAR(F'(I),S'(I))	CORR(F'(I),S'(I))
1	1	-0.000088919	-0.361851499	-0.000076314	-0.117027008
2	2	-0.000196198	-0.558426777	-0.000071249	-0.102512172
3	3	-0.000163993	-0.577387812	-0.000116082	-0.119292194
4	4	-0.000242920	-0.627164979	-0.000079342	-0.100855787
5	5	-0.000405113	-0.758253173	-0.000143095	-0.082271829

I	YR	COVAR(S'(I),S(I))	CORR(S'(I),S(I))	COVAR(S'(I),S(I+1))	CORR(S'(I),S(I+1))
1	1	0.000808850	0.272324296	-0.001110225	-0.306488921
2	2	0.001666550	0.366011409	-0.001462537	-0.330182605
3	3	0.001968865	0.383562160	-0.001693755	-0.365993881
4	4	0.001260906	0.325647315	-0.001422088	-0.225829192
5	5	0.005489554	0.431890425		

I	YR	COVAR(S'(I),F(I+1))	CORR(S'(I),F(I+1))
1	1	-0.000076738	-0.294634009
2	2	-0.000170115	-0.443238077
3	3	-0.000168268	-0.455269810
4	4	-0.000215694	-0.461125034
5	5	-0.000456106	-0.569585954

THE ABOVE ARE ESTIMATES OF THE SAMPLING COVARIANCES AND CORRELATIONS BETWEEN THE PARAMETER ESTIMATORS.

COVAR($\bar{S}$,$\bar{F}$) = -0.000023781 CORR($\bar{S}$,$\bar{F}$) = -0.501178877

COVAR($\bar{S}$',$\bar{F}$')= -0.000016203 CORR($\bar{S}$',$\bar{F}$')= -0.090023922

MATRIX OF DATA VALUES -- ADULTS

1	116.00	32.00	46.00	23.00	36.00	9.00
2	0.0	80.00	64.00	39.00	59.00	23.00
3	0.0	0.0	125.00	50.00	48.00	24.00
4	0.0	0.0	0.0	102.00	62.00	34.00
5	0.0	0.0	0.0	0.0	153.00	45.00
6	0.0	0.0	0.0	0.0	0.0	80.00

MATRIX OF EXPECTED VALUES -- ADULTS

1	116.00	43.06	48.11	23.49	23.12	8.23
2	0.0	78.16	87.31	42.63	41.96	14.93
3	0.0	0.0	115.43	56.36	55.47	19.74
4	0.0	0.0	0.0	84.81	83.48	29.70
5	0.0	0.0	0.0	0.0	146.04	51.96
6	0.0	0.0	0.0	0.0	0.0	80.00

MATRIX OF STANDARD NORMAL DEVIATES -- ADULTS

1	0.00	-1.71	-0.31	-0.10	2.70	0.27
2	0.0	0.21	-2.57	-0.56	2.67	2.10
3	0.0	0.0	0.93	-0.86	-1.02	0.97
4	0.0	0.0	0.0	1.92	-2.42	0.80
5	0.0	0.0	0.0	0.0	0.61	-0.98
6	0.0	0.0	0.0	0.0	0.0	0.00

MATRIX OF DATA VALUES -- YOUNG

1	161.00	34.00	21.00	7.00	18.00	4.00
2	0.0	109.00	73.00	12.00	16.00	7.00
3	0.0	0.0	171.00	51.00	26.00	13.00
4	0.0	0.0	0.0	157.00	57.00	10.00
5	0.0	0.0	0.0	0.0	198.00	39.00
6	0.0	0.0	0.0	0.0	0.0	102.00

MATRIX OF EXPECTED VALUES -- YOUNG

1	161.00	24.78	27.68	13.51	13.30	4.73
2	0.0	109.00	50.47	24.64	24.26	8.63
3	0.0	0.0	171.00	38.55	37.95	13.50
4	0.0	0.0	0.0	157.00	49.42	17.58
5	0.0	0.0	0.0	0.0	198.00	39.00
6	0.0	0.0	0.0	0.0	0.0	102.00

MATRIX OF STANDARD NORMAL DEVIATES -- YOUNG

1	0.0	1.88	-1.29	-1.78	1.30	-0.34
2	0.0	0.0	3.25	-2.58	-1.70	-0.56
3	0.0	0.0	0.0	2.04	-1.98	-0.14
4	0.0	0.0	0.0	0.0	1.11	-1.82
5	0.0	0.0	0.0	0.0	0.0	0.00
6	0.0	0.0	0.0	0.0	0.0	0.0

Example 4.2c

SYNTHETIC DATA GENERATED FOR 3 AGE CLASSES WHERE ADULTS & SUBADULTS ARE GROUPED

CHI-SQUARE TEST OF HO VS H1

	2 X 2 CONTINGENCY TABLE R(I.) N(I)-R(I.) / Q(I.) M(I)-Q(I.)		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM	2 X 2 CONTINGENCY TABLE M(I) Z(I+1) / Q(I,I) Q(I.)-Q(I,I)		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
I= 1	262	1238		116	146	
	245	755	18.359	161	84	23.480
I= 2	265	1235		146	349	
	217	783	6.272	109	108	28.217
I= 3	247	1253		329	375	
	261	739	34.390	171	90	26.910
I= 4	198	1302		284	379	
	224	776	36.195	157	67	49.746
I= 5	198	1302		475	169	
	237	763	46.026	198	39	9.200
I= 6	80	1420				
	102	898	21.053			

|TOTAL CHI-SQUARE WITH 11 DEGREES OF FREEDOM = 299.847|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 299.85 = 0.0

THIS TEST OF THE HYPOTHESIS HO AGAINST THE HYPOTHESIS H1 TESTS THE ASSUMPTION THAT YOUNG AND ADULTS HAVE THE SAME SURVIVAL AND RECOVERY RATES.

CHI-SQUARE TEST OF H1 VS H2

	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
I= 2	80	185	
	66	164	0.132
I= 3	125	122	
	204	253	2.294
I= 4	102	96	
	182	283	8.685
I= 5	153	45	
	322	124	1.825

|TOTAL CHI-SQUARE WITH 4 DEGREES OF FREEDOM = 12.937|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 12.94 = 0.01159

THIS TEST OF THE HYPOTHESIS H1 AGAINST THE HYPOTHESIS H2 TESTS THE ASSUMPTION THAT RECOVERY RATES FOR NEWLY RELEASED ADULTS ARE THE SAME AS FOR SURVIVORS OF PREVIOUSLY BANDED COHORTS.

CHI-SQUARE TEST OF H2 VS H3

	2 X 2 CONTINGENCY TABLE		CORRESPONDING CHI-SQUARE STATISTIC WITH 1 DEGREE OF FREEDOM
I= 1	34	50	
	32	114	8.975
I= 2	73	35	
	131	218	30.151
I= 3	51	39	
	131	244	14.392
I= 4	57	10	
	265	114	6.513

|TOTAL CHI-SQUARE WITH 4 DEGREES OF FREEDOM = 60.032|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 60.03 = 0.0

REJECTION OF H2 IN FAVOR OF H3 WOULD INDICATE THAT SURVIVAL AND RECOVERY RATES ARE AGE-DEPENDENT FOR AT LEAST THE FIRST TWO YEARS.

THE HYPOTHESES H01 AND H02 ARE MORE RESTRICTIVE THAN H1 BUT ARE NOT COMPARABLE IN THIS SENSE WITH HO, THUS H01 AND H02 DO NOT FIT INTO THE ABOVE SERIES. IN PRACTICE THE MODELS UNDER H02 AND H1 ARE LIKELY TO BE OF MOST USE, SO A LIKELIHOOD RATIO TEST TO DISTINGUISH BETWEEN THESE MODELS IS COMPUTED BELOW. THIS TESTS THE ASSUMPTION THAT YOUNG AND ADULT SURVIVAL RATES ARE CONSTANT FROM YEAR TO YEAR.

LIKELIHOOD RATIO TEST OF H02 VS H1.

CHI-SQUARE VALUE = 11.15
DEGREES OF FREEDOM = 8
PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN 11.15 = 0.19347

Example 4.2d

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SYNTHETIC DATA GENERATED FOR 3 AGE CLASSES WHERE ADULTS & SUBADULTS ARE GROUPED

CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H3
-------------------------------------------------------

                   CONTINGENCY TABLES                                        CORRESPONDING CHI-SQUARE
                                                                             STATISTICS AND DEGREES OF FREEDOM
                   ------------------                                        ---------------------------------

        I= 2
                   23. 39. 59. 64.
                    4.  7. 18. 21.
                    9. 23. 36. 46.                                              3.89 WITH  6 D.F.

        I= 3
                   24. 50. 48.
                    7. 12. 16.
                   36. 69.113.                                                  5.09 WITH  4 D.F.

        I= 4
                   34. 62.
                   13. 26.
                   67.177.                                                      2.29 WITH  2 D.F.

                                         |---------------------------------------|
                                         |TOTAL CHI-SQUARE       11.28 WITH 12 D.F.|
                                         |---------------------------------------|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN   11.28  = 0.50497



CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H0
-------------------------------------------------------

                                         |---------------------------------------|
                                         |TOTAL CHI-SQUARE      384.10 WITH 31 D.F.|
                                         |---------------------------------------|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN  384.10  = 0.0


CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H1
-------------------------------------------------------

                                         |---------------------------------------|
                                         |TOTAL CHI-SQUARE       84.25 WITH 20 D.F.|
                                         |---------------------------------------|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN   84.25  = 0.0


CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H2
-------------------------------------------------------

                                         |---------------------------------------|
                                         |TOTAL CHI-SQUARE       71.31 WITH 16 D.F.|
                                         |---------------------------------------|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN   71.31  = 0.00000


CHI-SQUARE GOODNESS OF FIT TEST OF THE MODEL UNDER H3
-------------------------------------------------------

                                         |---------------------------------------|
                                         |TOTAL CHI-SQUARE       11.28 WITH 12 D.F.|
                                         |---------------------------------------|

PROBABILITY OF A CHI-SQUARE VALUE LARGER THAN   11.28  = 0.50497

  FROM THE MODELS ABOVE, ONE SHOULD CHOOSE THE SIMPLEST MODEL (FEWEST PARAMETERS) THAT ADEQUATELY DESCRIBES THE DATA.
ADEQUACY MAY BE JUDGED BY EXAMINING THE RESULTS OF (1) THE GOODNESS OF FIT TESTS, AND (2) THE TESTS BETWEEN SPECIFIC
MODELS. FREQUENTLY, H02 OR H1 IS ADEQUATE.
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