Chapter 9. Planning a Banding Study

9.1 General Remarks

In this chapter we will suggest a number of considerations in planning a banding study. The remarks made here apply to other animal marking experiments although we make specific reference to bird banding. The following material represents only guidelines because value judgements are required in planning a banding study. We can rarely give specific and rigid details because so many factors are involved.

Band recovery data represent a sample from a banded population. Banding is done so that we have marked members of the population to sample at later dates. Banding studies are merely sampling experiments. In the case of exploited species of migratory birds, we have a very large number of "samplers" over a large geographic area. Consideration must be given to banding only birds that are representative of the total population of interest. If we wish to make inferences about the adult male redhead *(Aythya americana)* population in northern Utah from sampling and analysis of banded members, we must be sure the banded sample is representative of the adult male redheads in northern Utah. This rules out, for example, the use of sick, injured, transported, or color-marked birds. We further discuss representativeness in the following section.

Although bird banding is an expensive and time-consuming effort, insufficient attention has often been given to planning the study, assessing the assumptions, and thoroughly analyzing the data. The literature emphasizes the need for detailed planning of banding operations before the field work is initiated (DeLury 1947, 1951; Orians 1958). Data gathering and data analysis should be coordinated through proper planning and a realistic evaluation of the necessary assumptions. Planning is crucial if the study is to produce accurate and precise estimates of population parameters. As DeLury (1954: 293) pointed out, "...it is an expensive impropriety to maintain that an untrustworthy estimate is better than none." **Review of published material by Cormack (1979) and Seber (1972, 1982)** as well as this handbook is recommended.

For the bird bander or field biologist the primary drawback in the use of the newer techniques is likely to be their complexity. The theoretical basis of most of the better methods lies deep in the field of mathematical statistics. In most situations the use of a particular method is fairly straightforward, but the mathematical theory and derivation of the various estimators may seem difficult. We have included examples illustrating the use of the methods we describe. However, it is important for the bird bander or field biologist to fully understand the assumptions and limitations of these methods as well as their strengths. This suggests that a joint effort between biologists, bird banders, and ornithologists on the one hand, and statisticians on the other, would be advantageous. This combination of talent and expertise would be particularly advantageous for comprehensive analyses of regional or flyway banding programs.

The advantages of large-scale cooperative banding programs are obvious. Entire populations can be sampled including population segments in various geographic areas. Indeed, continental, national, and regional banding programs are underway at this time for several migratory species (e.g., mallard, Canada geese, mourning dove *(Zenaidura macroura)*, wood duck, and band-tailed pigeon *(Columba fasciata)*). The results of such cooperative banding programs can address major management and research needs. More comprehensive objectives and difficult questions and objectives can be approached when parameter estimates and test statistics are available from numerous banded segments of widespread populations (e.g., Anderson 1975 and Anderson and Burnham 1976). Of course, much can be learned from single banding programs aimed at one segment of the population (e.g., Canada geese wintering in southern Illinois). In any event, the computer programs described by Cowardin (1977) and Davenport (1977) may be useful.

In summary, several points could be emphasized. Bird banding studies must be developed on a scientific basis if accurate and precise results are to be expected. Planning the project and reviewing the literature should certainly precede the field work. Estimates of various population parameters are not made easily and should not be made casually. Assumptions underlying the method of analysis to be used should be fully recognized. The newer stochastic models have a number of important advantages and therefore should replace the life table approaches. Estimates of the sampling variances of parameter estimates are of great importance in bird banding studies. Estimates of population parameters, made after years of expensive field work, that have extremely wide confidence intervals represent wasted time and money.

9.2 Suggestions – Guidelines

Banding of birds may appropriately be done at various times of the year depending on objectives and availability of birds. In general, we recommend that game birds be banded just before or just after the hunting season.

Populations are usually sedentary during these periods and often large samples can be banded. Banding during migration periods is generally to be discouraged because it is nearly impossible to determine which population the estimates relate to. Banding during the breeding season is often quite disruptive to the population; however, there are many species where only breeding-season banding is feasible. Banding during the hunting season is also not recommended. However, there are instances where large samples of birds (e.g., Canada geese) can be banded only during the latter part of the hunting season. Hunting often keeps birds concentrated on refuge areas where they can be banded. The birds disperse rapidly as soon as the hunting season terminates and become difficult to band in large numbers. In some instances, Models 0 and H_2 can be used as an approximation for in-season banding data (see Sections 2.5 and 3.5). Banding programs for nongame birds should probably follow the same general periods: late summer-early fall, winter, and (for some species) during the breeding seasons.

The actual banding period should be short in relation to the period between bandings (survival period). Mortality during the banding period is undesirable. Annual banding during a 3- to 5-week period is probably satisfactory if the survival period is 12 months. In some cases mourning doves are sometimes banded over a 2- to 4-month period which is likely to be unsatisfactory if the objective relates to the estimation of population parameters. Lengthy banding periods are especially undesirable for species with a high annual mortality rate.

A very important remark concerns studies involving large banded samples of young birds without an accompanying sample of adult birds. Unless some very restrictive and unrealistic assumptions are made no satisfactory estimates of survival rates are possible (Section 3.9). In general, the use of data from banded young birds will produce seriously biased parameter estimates (similar to the large bias associated with life table methods). Unfortunately, many of the studies of gulls, terns, and other colonial nesting birds fall into this category. It is a simple matter to band young birds in the colony while still in the nest, but obtaining a sample of adults has proven very difficult and quite disruptive to the colony. Seber (1972) and Cormack (in an appendix to Fordham 1970) have developed ML methods for age-specific survival rate if it can be assumed that reporting/recovery rates are constant and independent of age and year and that survival is time-independent. These assumptions are unlikely to be supported by biological data. In addition, both authors acknowledge problems with the performance of their method. *We expect few if any new developments in methodology that will lessen the need to band a matching sample of adults.*

Differences in survival and recovery rates by sex seem to be very common. We provide a test to examine this question (Section 5.1). Generally, separate analyses should be carried out for males and females, although we have seen several sets of Canada goose data where male and female survival and recovery rates are essentially identical (this might be expected for a species that is not obviously sexually dimorphic and migrates in loose family groups). In these situations, the data for males and females (numbers banded and the recovery arrays) can be pooled before the analysis and parameter estimation takes place. This appears to be the exception rather than the rule, however, suggesting that sample sizes required for a given level of precision (Section 9.3) must be estimated separately for males and females.

Except for Section 8.2, this handbook focuses on methods for analyzing recovery data. Several very good ML methods for the analysis of live recaptures are now in the literature (Jolly 1965, Pollock 1979, Jolly 1979, Pollock 1981b, Buckland 1980, Brownie and Robson 1983, Stokes 1984, Crosbie and Manly, 1985). These methods deal with generalized Jolly-Seber models (e.g., age-specific parameters) as well as restricted models wherein some parameters are the same over time. Also, there are some good computer programs specifically for analysis of live recapture data (Aranson and Baniuk 1980, Jolly and Dickson 1980, Nichols et al. in press, Crosbie and Manly 1985). It is now possible to do a fully efficient analysis of live recaptures and dead recoveries when both appear in the same data set. Buckland (1980) discusses one way to do this, however we feel that often the parameterization for the two types of data, coded into separate [m_{ij}] arrays (See Section 8.2), must be different. While no specific software for such an analysis exist, program SURVIV (White 1983) can be used to estimate survival rates, and test if these rates differ between the data sets from live vs. dead samples. This has many important and interesting possibilities for studies such as those reported by Anderson and Sterling (1974). For these reasons, we recommend that recaptures should be recorded as part of bird banding programs in the field. That is, birds captured alive in year i from banding at the same site in previous years should be carefully recorded. Of course, with nongame birds, recapture information is often the only type of data available, and the number of recaptures are often small. Analysis of such sparse recapture data has been considered by Kreger (1973) and Mardekian and McDonald (1981).

We have explained that the theory underlying many modern methods allows the periods between banding to be unequal, but most analyses and tests carried out by the computer programs are valid only if these periods are equal. In general, we recommend banding studies to be based on annual periods. We also question studies where birds are banded every 2 or 3 years. Although there is probably nothing fundamentally wrong with such studies, we question what objective is being satisfied by such programs.

Marking of birds (e.g., dyeing of feathers or use of patagial tags and ribbons, colored plastic leg bands, neck collars, nasal saddles, etc.) is generally done toward objectives other than estimating vital parameters. Hunters and others are much more likely to shoot and/or report a vivid pink snow goose *(Chen hyperborea)* with a colored neck collar than a normal snow goose. Therefore, if estimates of survival rates are of interest, these practices must be avoided. However, if the mark disappears within a year (e.g., molt of the dyed feathers, etc.) Models 0 or H_2 may be appropriate and thus allow the banding and recovery data to be used effectively toward two divergent objectives. Most common objectives would be best served by banding and analyzing only normal wild birds caught, banded, and released immediately, uninjured. Studies of game birds often use only recoveries reported as shot during the hunting season. This practice allows $\hat{f}_i$ to be used as an index to harvest rate. If the sole objective is to estimate survival rates, then nearly all recoveries can be used, thereby increasing sample size and precision.

The use of Reward bands (cf. Henny and Burnham 1976) has several substantial advantages in a few studies. This technique must be used with care. The advantages are primarily that an estimate of harvest rate is obtained, rather than merely an index to harvest rate. If samples of Reward and regular bands are used, an estimate of the band reporting rate can be made and thereby allow past data to be interpreted more fully. The use of Reward bands increases the number of recoveries, often by a factor of 2 or 3, given a fixed number of birds banded.

We believe banding programs should be conducted for a minimum of 5 years ($k \geq 5$). This will allow at most four estimates of survival. It takes several years to get a good-sized population of banded birds in the total population. Programs in operation for less than 5 years are not likely to produce useful estimates. Because precision is gained when $\ell > k$, the investigator may often wish to reanalyze the accumulated recovery data 2-4 years after banding is terminated to make a "final" analysis of the data. This allows the use of the additional recoveries from later years to be incorporated in the analysis.

Species having low band recovery rates are particularly troublesome. For example, adult blue-winged teal and American woodcock typically have recovery rates of about 1%. If 1,000 were banded in August of year i we would only expect to get 10 reported the first year after banding, and perhaps only 20 over the total life-span of the banded cohort. These figures are totally insufficient so we might have to envision banding 3 or 4 thousand each year to obtain estimates of annual survival that are even reasonably precise. Usually, samples of this size are not obtainable, particularly for each age and sex. One may ask if the 1,000 should be banded anyway and hope to obtain useful parameter estimates and test statistics concerning the population. Our research suggests the answer to this question is no. Johnson (1974) discusses a hypothetical example where small samples of birds were banded over a 3-year period with resulting parameter estimates having very wide confidence intervals at best. He questions the value of such a practice. The important question of how many birds should be banded annually to obtain a specified level of precision is discussed in the next section.

Needless to say, incorrect aging and/or sexing of birds in the field at the time of banding has devastating effects on the study. We have seen some examples of such problems, for example, in mourning dove data. Many songbirds are banded without identifying either the age or the sex. Data from such studies cannot be used for estimating vital statistics but perhaps some distributional data will be obtained.

The subject of "representativeness" could well occupy a chapter of its own. Consider the large-scale duck banding program conducted annually in the northern two-thirds of North America. Typically birds are banded on concentration areas (staging areas). Here, they are easily trapped and banded in sufficient numbers. It could be asked if the birds in singles or pairs or small groups on the smaller water areas might represent a different population segment and, since they are less likely to be banded, inferences from the banded sample to the entire population would not be valid. We suggest that this is not especially important because the recovery area is quite large (e.g., the major part of the Central and Mississippi Flyways) and both "types" of birds pass over the recovery (sampling) area during the fall migration. This suggests that the location of the birds during the banding period in late August will have little relevance to the sampling scheme (hunting). This conclusion might change if most hunting deaths took place on the staging areas and the birds remained on these areas during the hunting season. Neither of these conditions seems to be true, at least with migratory species.

The above example suggests we can obtain our sample by banding the birds in a representative manner, or by employing a representative recovery (sampling) procedure after a random mixing process of the birds. These considerations might be quite different with studies of resident species, either game or nongame.

In planning new banding studies of resident birds we urge that biologists consider biannual (pre- and post-hunting season) banding programs: Models H_7 and H_8 (Chapter 7). These models may allow a study of the effect of hunting on the survival process if adequate data are collected. Here again we emphasize the possible use of Reward bands in such studies. A second potential lies in data gathering efforts for Models H_4, H_5, and H_6. These models are appropriate for those species that can be identified and banded according to the three age classes at the time of banding. Furthermore, annual samples of sufficient size must be obtained for each age group. The potential for this type of study is large, however, and the data requirements are stringent, perhaps impossible except for a few species.

In summary we emphasize that banding is not a panacea. Often in migratory bird management and research the word "study" has been nearly synonymous with the word "band." Banding is an excellent tool for *some* species for *some* objectives. It is possible to obtain some basic estimates of vital statistics of the population and gain insight into some very fundamental relationships concerning population dynamics. For some studies aimed at certain objectives, banding is usually the only method we now have. In some situations, no alternative exists because we have no other means to obtain needed information. These are the situations for which the material in this handbook will be useful. Unfortunately, many species cannot easily be banded or have behavioral habits that defy banding in sufficient numbers. In these respects, banding is no different than any other tool or technique. It is useful if used intelligently in the proper situation.

9.3 Sample Size Guidelines for Adults

The General Approach

In planning a banding study one question sure to arise is how many birds to band. The variables to be specified are the annual numbers of adults to band, $N_1, \ldots, N_k$ and k, the number of years of banding. From experience in analyzing real data and knowledge of these models and methods, we suggest 300 adults per year as a minimum for any species, even those with high recovery rates (for species with low recovery rates, more than 300 birds a year must be banded for reliable results). Otherwise the hypothesis tests and confidence intervals computed will be unreliable as they require "large samples." As mentioned above we recommend at least 5 years of banding as a minimum goal when planning a banding study. Finally, it is reasonable in planning the study to aim for equal numbers of banded birds each year. Thus, in what follows we assume $N_1 = N_2 = \cdots = N_k \equiv N$.

Specific sample size considerations require that we have a criterion to be satisfied by our choice of N. Consequently, we assume it is desired to estimate the average survival $\bar{S}$, using Model 1, within a specified precision after the recoveries are available for all k years of the study. An alternate approach, which has been developed by Youngs and Robson (1975), is to determine each annual banding quota (N_i) to achieve specific precision for each annual survival rate estimator $\hat{S}_i$. That is, their paper gives the methodology to determine N_i, given a desired variance $\text{VAR}(\hat{S}_i)$ for any (or all) years. The reader who wishes to specify each N_i separately in this way is referred to Youngs and Robson (1975).

Let $\text{VAR}(\tilde{\bar{S}})$ be the sampling variance of $\bar{S}$ (see Section 2.2), and $\text{SE}(\tilde{\bar{S}}) = \sqrt{\text{VAR}(\tilde{\bar{S}})}$ be the corresponding standard error of $\tilde{\bar{S}}$. An approximate 95% confidence interval on $\bar{S}$ is given by $\tilde{\bar{S}} \pm 1.96\ \text{SE}(\tilde{\bar{S}})$. This formula can be rewritten as $\tilde{\bar{S}}(1. \pm 1.96\ \text{CV}(\tilde{\bar{S}}))$, where $\text{CV}(\tilde{\bar{S}})$ is the coefficient of variation of the estimator $\tilde{\bar{S}}$, $\text{CV}(\tilde{\bar{S}}) = \text{SE}(\tilde{\bar{S}}) / \bar{S}$.

A convenient mathematical criterion for choosing the sample size N is to have $\text{CV}(\tilde{\bar{S}})$ less than or equal to some specified value (e.g., 0.05 or 0.1). After k years, we can then estimate average annual survival within a preset level of precision. This is equivalent to aiming for a confidence interval on $\bar{S}$ with a specified expected width.

The true coefficient of variation of $\tilde{\bar{S}}$ depends upon the unknown annual rates S_i and f_i that will apply during the study, and upon N and k. Exact sample size determination is thus impossible. A practical course of action is to obtain the best guess of the anticipated average rates $\bar{S}$ and $\bar{f}$, then use them in place of the unknown S_i and f_i in the formula for $\text{CV}(\tilde{\bar{S}})$. These anticipated average rates can usually be deduced from available data on the same or related species.

It will be shown below that $\text{VAR}(\tilde{\bar{S}})$, and hence $\text{CV}(\tilde{\bar{S}})$, is proportional to $1/N$. In fact once we replace S_i and f_i by $\bar{S}$ and $\bar{f}$, respectively, in the formula for $\text{VAR}(\tilde{\bar{S}})$, then the expression

$$[\text{CV}(\tilde{\bar{S}})]^2 = \frac{h(\bar{S}, \bar{f}, k)}{N} \tag{9.3.1}$$

results, where the function $h(\cdot)$ depends only upon $\bar{S}, \bar{f}$, and k.

The steps for sample size determination on adults are as follows:

(1) Specify k and the desired relative precision $\text{CV}(\tilde{\bar{S}})$;

(2) supply the best known values of $\bar{S}, \bar{f}$; and

(3) compute $h(\bar{S}, \bar{f}, k)$ and from (9.3.1) obtain N as $N = h(\bar{S}, \bar{f}, k) / [\text{CV}(\tilde{\bar{S}})]^2$.

We will exhibit the formula for $h(\cdot)$ below, and then give a simple way to compute it.

First a word about choosing $\text{CV}(\tilde{\bar{S}})$. We recommend $\text{CV}(\tilde{\bar{S}}) \leq 0.05$, or smaller. If this coefficient of variation were as large as 0.1, with for example, $\bar{S} = 0.5$, the expected 95% confidence interval on $\bar{S}$ would be (0.4, 0.6). Such a wide interval is not very useful. For the same value of $\bar{S} = 0.5$, a value of $\text{CV}(\tilde{\bar{S}}) = 0.05$ gives an expected 95% confidence interval of (0.45, 0.55). If a very precise estimator of $\bar{S}$ is desired, a value of $CV(\tilde{\bar{S}}) = 0.025$ or less would not be unreasonable as a goal.

The Function $h(\bar{S}, \bar{f}, k)$

The sampling variance of $\tilde{\bar{S}}$ is expressed, in general, as

$$\text{VAR}(\tilde{\bar{S}}) = \left[\sum_{i=1}^{k-1} \text{VAR}(\tilde{S}_i) + 2 \sum_{i=1}^{k-2} \sum_{j=i+1}^{k-1} \text{COV}(\tilde{S}_i, \tilde{S}_j) \right] \Big/ (k-1)^2 . \tag{9.3.2}$$

From Robson and Youngs (1971), formulas for the theoretical (large sample) sampling variances and covariances needed above are

$$\text{VAR}(\tilde{S}_i) = (S_i)^2 \left[\frac{1}{\text{E}(R_i)} - \frac{1}{N_i} + \frac{1}{\text{E}(R_{i+1})} - \frac{1}{N_{i+1}} + \frac{1}{\text{E}(T_i - C_i)} - \frac{1}{\text{E}(T_i)} \right] \quad , i = 1, \ldots, k-1$$

and

$$\text{COV}(\tilde{S}_i, \tilde{S}_j) = \begin{cases} 0 & , j > i+1 \\ -(S_i S_{i+1}) \left[\dfrac{1}{\text{E}(R_{i+1})} - \dfrac{1}{N_{i+1}} \right] & , j = i+1, \ i = 1, \ldots, k-2 . \end{cases}$$

The notation of Chapter 2 is used above; for example, R_i is the i^{th} row total of the recovery data array.

If we use the restriction of equal N_i in all years, the above variances and covariances are all proportional to $1/N$. This is because terms like $\mathbf{E}(R_i)$, $\mathbf{E}(T_i - C_i)$ and $\mathbf{E}(T_i)$ are all directly proportional to N. For example

$$\mathbf{E}(R_i) = N(f_i + S_i f_{i+1} + \cdots + S_i \cdots S_{k-1} f_k) .$$

Replacing the annual rates S_i and f_i, implicitly involved in the formulae for $\text{VAR}(\tilde{S}_i)$, by $\bar{S}$ and $\bar{f}$ allows us to write the approximations we must use in practice for sample size determination

$$\text{VAR}(\tilde{S}_i) = \frac{(\bar{S})^2}{N} \left[\frac{1}{R_i^*} + \frac{1}{R_{i+1}^*} + \frac{1}{T_i^* - C_i^*} - \frac{1}{T_i^*} - 2 \right] \tag{9.3.3}$$

$$\text{COV}(\tilde{S}_i, \tilde{S}_j) = \begin{cases} 0 & , j > i+1 \\ -\dfrac{(\bar{S})^2}{N} \left[\dfrac{1}{R_{i+1}^*} - 1 \right] & , j = i+1 . \end{cases} \tag{9.3.4}$$

The terms R_i^*, C_i^*, T_i^* are functions only of $\bar{S}$, $\bar{f}$ and k. They are easily computed by constructing the model structure in terms of the cell probabilities $\pi_{ij} = (\bar{S})^{j-i} \bar{f}$, treating these probabilities as data and applying the formulae for R_i, C_i and T_i to compute R_i^*, C_i^* and T_i^*. Specific formulae are

$$R_i^* = \sum_{j=i}^{k} \pi_{ij} = \bar{f} \left(\frac{1 - (\bar{S})^{k-i+1}}{1 - \bar{S}} \right) \quad , i = 1, \ldots, k ,$$

$$C_i^* = \sum_{i=1}^{j} \pi_{ij} = \bar{f} \left(\frac{1 - (\bar{S})^i}{1 - \bar{S}} \right) \quad , i = 1, \ldots, k ,$$

and

$$T_i^* = T_{i-1}^* - C_{i-1}^* + R_i^* \quad , i = 2, \ldots, k ,$$

with $T_1^* \equiv R_1^*$. Explicitly

$$T_i^* = \bar{f} \left(\frac{1 - (\bar{S})^{k-i+1}}{1 - \bar{S}} \right) \left(\frac{1 - (\bar{S})^i}{1 - \bar{S}} \right) , i = 1, \ldots, k .$$

Using formulae (9.3.2), (9.3.3) and (9.3.4) gives, after simplification,

$$(k-1)^2 \, \text{VAR}(\tilde{\bar{S}}) = \frac{(\bar{S})^2}{N} \left[\sum_{i=1}^{k-1} \left(\frac{1}{T_i^* - C_i^*} \right) - \sum_{i=1}^{k-1} \left(\frac{1}{T_i^*} \right) + \frac{1}{R_1^*} + \frac{1}{R_k^*} - 2 \right] .$$

From $[CV(\tilde{\bar{S}})]^2 = \text{VAR}(\tilde{\bar{S}}) / (\bar{S})^2$, and the above formula, we derive $h(\bar{S}, \bar{f}, k)$ of formula (9.3.1) as

$$h(\bar{S}, \bar{f}, k) = \frac{\left[\sum_{i=1}^{k-1} \left(\frac{1}{T_i^* - C_i^*} \right) - \sum_{i=1}^{k-1} \left(\frac{1}{T_i^*} \right) + \frac{1}{R_1^*} + \frac{1}{R_k^*} - 2 \right]}{(k-1)^2} , \tag{9.3.5}$$

where as shown above T_i^*, C_i^*, R_i^*, and R_i^* depend only upon $\bar{S}$, $\bar{f}$, and k. Unfortunately, (9.3.5) does not appreciably simplify.

If the terms R_i^*, C_i^*, and T_i^* are written out as functions of $\bar{S}$ and $\bar{f}$, one sees that $h(\cdot)$ is approximately proportional to $1/\bar{f}$ times a function of only $\bar{S}$,k. Consequently, to a first-order approximation the required sample size is proportional to $1/\bar{f}$. This implies a given precision could be met, for example, with half the banded sample if $\bar{f}$ could be doubled. It is worth emphasizing that average recovery rate is one of the most important controlling parameters affecting the precision of estimates of S and f from banding studies. Studies on species with very low recovery rates are usually pointless, unless huge samples can be banded annually.

Computing the Function $h(\bar{S}, \bar{f}, k)$

Given a study length k, a specified precision on $\bar{\hat{S}}$, via the CV$(\bar{\hat{S}})$, and reasonable guesses for $\bar{S}$ and $\bar{f}$, then the required value of N is computed by finding $h(\bar{S}, \bar{f}, k)$ and applying

$$N = \frac{h(\bar{S}, \bar{f}, k)}{[\mathrm{CV}(\bar{\hat{S}})]^2} \ . \tag{9.3.6}$$

The function $h(\cdot)$ given in (9.3.5) is easily calculated.

First generate the triangular array of cell probabilities ("data") $\pi_{ij} = (\bar{S})^{j-i}\bar{f}$.

Year banded	Probability of band recovery in year j: 1	2	3	,...,	k	Row totals
1	$\bar{f}$	$\bar{S}\bar{f}$	$\bar{S}^2\bar{f}$	,...,	$\bar{S}^{k-1}\bar{f}$	R_1^*
2		$\bar{f}$	$\bar{S}\bar{f}$	,...,	$\bar{S}^{k-2}\bar{f}$	R_2^*
3			$\bar{f}$	,...,	$\bar{S}^{k-3}\bar{f}$	R_3^*
.				.	.	.
.				.	.	.
.				.	.	.
k					$\bar{f}$	R_k^*
Column totals	C_1^*	C_2^*	C_3^*	,...,	C_k^*	

It is necessary to generate only the first row of the array; subsequent rows are merely truncated versions of row 1. Then sum the array rows and columns to obtain R_i^*, C_i^*. The mathematically inclined person can easily short-cut this process as the relationship $R_i^* = C_{k-i+1}^*$ holds.

Next the values of T_i^* are computed from the formula $T_i^* = R_i^* \, C_i^* / \bar{f}$. Finally, it is easy to compute $T_i^* - C_i^*$. It will be convenient to arrange these numbers in a brief table, computing the columns in the order shown, for $i = 1, \ldots, k-1$,

$$i \qquad R_i^* \qquad C_i^* \qquad T_i^* \qquad T_i^* - C_i^* \, .$$

Then compute three summary statistics from this table:

$$A = \sum_{i=1}^{k-1} \frac{1}{T_i^* - C_i^*} \, ,$$

$$B = \sum_{i=1}^{k-1} \frac{1}{T_i^*} \, ,$$

$$C = \frac{1}{R_1^*} + \frac{1}{\bar{f}} - 2 \, .$$

Items A and B are the sums of reciprocals of elements in columns 4 and 3, respectively, of the table.

Finally

$$h(\bar{S}, \bar{f}, k) = \frac{A - B + C}{(k-1)^2} \, .$$

An Example

Assume a 5-year ($k=5$) study on adult male mallards, banded preseason, is planned. From past data (e.g., Anderson 1975), $\bar{S}=0.6$ and $\bar{f}=0.07$ are reasonable values to use. Now compute the table of estimated probabilities of band recovery:

Year banded	Probability of band recovery in year j				
	1	2	3	4	5
1	0.0700	0.0420	0.0252	0.0151	0.0091
2		0.0700	0.0420	0.0252	0.0151
3			0.0700	0.0420	0.0252
4				0.0700	0.0420
5					0.0700

Only row 1 requires calculation; even there, element one is simply $\bar{f}=0.07$ and succeeding elements are just $\bar{S}=0.6$ times the previous element. For example, 0.0420 = (0.6) (0.07) and 0.0252 = (0.6) (0.0420).

Next, sum the rows and columns to obtain the partial table:

i	R_i^*	C_i^*
1	0.1614	0.0700
2	0.1523	0.1120
3	0.1372	0.1372
4	0.1120	0.1523

Now compute the column for $T_i^* = R_i^* C_i^* / \bar{f}$. For example,

$$T_1^* = (0.1614)\,(0.0700)/(0.0700) = 0.1614,$$
$$T_2^* = (0.1523)\,(0.1120)/(0.0700) = 0.2437.$$

Once the T_i^* column is written down it is easy to write the fourth column $T_i^* - C_i^*$. The completed table is

i	R_i^*	C_i^*	T_i^*	$T_i^* - C_i^*$
1	0.1614	0.0700	0.1614	0.0914
2	0.1523	0.1120	0.2437	0.1317
3	0.1372	0.1372	0.2689	0.1317
4	0.1120	0.1523	0.2437	0.0914.

Now find the sums of the reciprocals of the last two columns. On most calculators this is easy. For the example

$$A = \sum_{i=1}^{4} \frac{1}{T_i^* - C_i^*} = \frac{1}{0.0914} + \frac{1}{0.1317} + \frac{1}{0.1317} + \frac{1}{0.0914} = 37.0679.$$

Similarly

$$B = \sum_{i=1}^{4} \frac{1}{T_i^*} = 18.1214,$$

and

$$C = \frac{1}{R_1^*} + \frac{1}{\bar{f}} - 2 = \frac{1}{0.1614} + \frac{1}{0.0700} - 2 = 18.4815.$$

Finally

$$h(0.6, 0.07, 5) = \frac{A - B + C}{4^2} = \frac{37.0679 - 18.1214 + 18.4815}{16} = 2.3393.$$

Up to this point it has not been necessary to specify a coefficient of variation on $\bar{\bar{S}}$. Assume CV$(\bar{\bar{S}}) = 0.04$ is specified; then from (9.3.6) the sample size needed (numbers to band *each* year for 5 years) is

$$N = \frac{2.3393}{[0.04]^2} = 1{,}462.$$

This is a reasonable number for a species like the mallard that is easy to trap and band. Thus, if about 1,500 adult male birds were banded each year for 5 years, then our expected 95% confidence interval around the mean survival rate (0.60) for the 5-year period would be 0.552 to 0.648 (the actual interval, being a random variable, would vary somewhat from these theoretical limits).

Other species are more difficult to capture and band; e.g., canvasback *(Aythya valisineria)* and snipe *(Capella gallinago)*. Let us now examine the situation if only 500 birds per year could be banded (assuming the same values for $\bar{S}$, $\bar{f}$, and k). Then, using the above value of $h(\cdot)$ and $N = 500$, we can compute the relative precision to be expected:

$$\text{CV}(\bar{\bar{S}}) = \sqrt{\frac{h(\bar{S},\bar{f},k)}{N}} = \sqrt{\frac{2.3393}{500}} = 0.0684.$$

In this case, the expected 95% confidence interval on the mean survival (0.60) is wider, 0.520 to 0.680. Of course, the 95% confidence intervals on the *individual* annual estimates of survival would be much wider than this.

This illustrates the point that these same calculations needed for sample size determination can be used to answer the question—"If we band N birds per year for k years, with $\bar{S}$ and $\bar{f}$ as anticipated parameters, what relative precision will we achieve?" We recommend these calculations be made in any study, whether to determine N, or determine CV$(\bar{\bar{S}})$ given N. If the indicated CV$(\bar{\bar{S}})$ (for N given) is more than 0.1, the study is essentially worthless if survival rate estimation is the primary objective.

We emphasize that this procedure gives only guidelines for choosing the annual sample size. It is dependent upon a number of assumptions: (1) Model 1 or 2 (not Models 0 or 3) is correct, (2) the chosen values of $\bar{S}$ and $\bar{f}$ are near the values that will apply during the study, and (3) only expected confidence limits are given; the actual limits computed after the study will not be identical to the predicted ones. The assumption $N = N_1 = \cdots = N_k$ is not critical, however. If actual numbers banded vary about the target value of N by even $\pm 10\%$, these sample size considerations will still by useful. Program ESTIMATE allows sample size guidelines to be computed as an option (See Chapter 6).

9.4 Sample Size Guidelines for Young

The General Approach

Adult birds should always be banded in conjunction with banding of young (e.g., see Section 3.9). Thus we are faced with determining sample sizes for adults as well as young when young are to be banded. Consequently, the reader should understand Section 9.3 before proceeding with this section.

In the problem at hand, the variables to be specified are the annual numbers of adults (N_i) and young (M_i) to band each year for k years. As with adults only, we recommend a minimum of 300 birds of each age class be banded each year. Also, we recommend 5 years of consecutive banding as a minimum goal for the study length if survival rate estimation is desired. It is reasonable in planning the study to aim for equal numbers of banded birds in each age class, each year; thus we assume $N_i \equiv N$ and $M_i \equiv M$ for $i = 1, \ldots, k$. Finally, in what follows we assume that Model H_1 will describe the band recovery data from the study, and therefore the notation of Chapter 3 will be used here.

From considerable experience with banding data we have observed that the precision of the adult survival rate estimators based on Model H_1 (adults and young) is not very different from the precision obtained with Model 1 (adults only). Consequently, we recommend that the adult sample size first be determined by the methods of Section 9.3. It will then be possible to determine an annual (target) sample size for numbers of young to band.

Let S_i' be the annual young survival rate. Assume it is desired to estimate $\bar{S}'$, the average annual young survival rate during the study, with a specified precision. Let VAR$(\bar{\bar{S}}')$ be the sampling variance of the estimator $\bar{\bar{S}}'$ and SE$(\bar{\bar{S}}') = \sqrt{\text{VAR}(\bar{\bar{S}}')}$ be the corresponding standard error of $\bar{\bar{S}}'$. An approximate 95% confidence interval on $\bar{S}'$ is given by $\bar{\bar{S}}' \pm 1.96$ SE$(\bar{\bar{S}}')$. This formula can be rewritten as $\bar{\bar{S}}(1. \pm 1.96\ \text{CV}(\bar{\bar{S}}'))$, where CV$(\bar{\bar{S}}')$ is the coefficient of variation of the estimator $\bar{\bar{S}}'$: CV$(\bar{\bar{S}}') = $ SE$(\bar{\bar{S}}') / \bar{S}'$.

As with adult sample size determination, a convenient mathematical criterion for choosing the sample size M is to have $CV(\bar{\tilde{S}}')$ less than or equal to some specified value, such as 0.05 or 0.025. This is equivalent to choosing a sample size such as that there is a prescribed expected confidence interval width on $\bar{S}'$.

The true coefficient of variation of $\bar{\tilde{S}}'$ depends upon the unknown annual survival and recovery rates (S_i', S_i f_i', and f_i) for young and adults that will apply during the study. Exact sample size determination is thus impossible. As in determination of adult sample size N, a practical course of action is to obtain the best guess of the anticipated average rates, $\bar{S}'$, $\bar{S}$, $\bar{f}'$, and $\bar{f}$, then use them in place of the appropriate unknown year-specific parameters in the formula for $CV(\bar{\tilde{S}}')$. These anticipated average rates can usually be deduced from available data on the same or related species.

Given the above approach to determining sample size for young, a formula for M can be derived. Given the requisite adult sample size has been computed, it is then easy to compute M. Derivation of the formula will be given below in a separate subsection; the less mathematical reader can skip this material.

Derivation of the Formula for M

From Brownie (1973:14)

$$\mathrm{VAR}(\tilde{S}_i') = (S_i')^2\left[\frac{1}{\mathbf{E}(Q_{i\cdot} - Q_{ii})} - \frac{1}{M_i} + \frac{1}{\mathbf{E}(R_{i+1\cdot})} - \frac{1}{N_{i+1}}\right] \tag{9.4.1.}$$

and

$$\mathrm{COV}(\tilde{S}_i', \tilde{S}_j') = 0 \qquad i \neq j, i = 1, \ldots, k-1.$$

It follows that

$$\mathrm{VAR}(\bar{\tilde{S}}') = \left(\frac{1}{k-1}\right)^2 \sum_{i=1}^{k-1} \mathrm{VAR}(\tilde{S}_i').$$

Upon making the assumptions $N_i \equiv N, M_i \equiv M, S_i' \equiv \bar{S}', S_i \equiv \bar{S}, f_i' \equiv \bar{f}'$ and $f_i \equiv \bar{f}$, explicit results can be derived:

$$\mathrm{VAR}(\bar{\tilde{S}}') = \left(\frac{1}{k-1}\right)^2 (\bar{S}')^2 \left[\sum_{i=1}^{k-1}\left\{\frac{1}{\mathbf{E}(Q_{i\cdot} - Q_{ii})} + \frac{1}{\mathbf{E}(R_{i+1\cdot})}\right\} - \frac{k-1}{M} - \frac{k-1}{N}\right],$$

$$[(k-1)CV(\bar{\tilde{S}}')]^2 = \frac{1}{M}\frac{1}{\bar{S}}\left[\sum_{i=1}^{k-1}\frac{1}{R_{i+1}^*}\right] + \frac{1}{N}\left[\sum_{i=1}^{k-1}\frac{1}{R_{i+1}^*}\right] - (k-1)\left[\frac{1}{M} + \frac{1}{N}\right]. \tag{9.4.2}$$

In formula (9.4.2) R_i^* has the same meaning as in Section 9.3; specifically it depends only upon adult parameters $\bar{S}$ and $\bar{f}$, not upon parameters for young. Because of the relationship between R_i^* and C_i^* formula (9.4.2) may be rewritten as

$$[(k-1)CV(\bar{\tilde{S}}')]^2 = \frac{1}{M}\left[\frac{A'}{\bar{S}'} - (k-1)\right] + \frac{1}{N}\left[A' - (k-1)\right],$$

where

$$A' = \sum_{i=1}^{k-1}\frac{1}{C_i^*}$$

and

$$C_i^* = \bar{f}\left(\frac{1-(\bar{S})^i}{1-\bar{S}}\right).$$

Finally the formula for M is

$$M = \frac{\left[\frac{A'}{\bar{S}'} - (k-1)\right]}{\left[(k-1)CV(\bar{\tilde{S}}')\right]^2 - \left[\frac{A' - (k-1)}{N}\right]}. \tag{9.4.3}$$

An Example and Some Considerations

The desired sample size M is given by formula (9.4.3), where the quantity A' is given by

$$A' = \sum_{i=1}^{k-1}\frac{1}{C_i^*},$$

with the C_i^* having exactly the same meaning here as in Section 9.3. Hence the C_i^* depend only upon adult parameter values, and must be computed anyway to determine the value of N which appears in formula (9.4.3).

The example of Section 9.3 will be elaborated upon here. Assume young are also to be banded, and that $\bar{S}'=0.5$ (note $\bar{f}'$ drops out of the final formula for M, hence need not be considered). In the preceding example $S=0.6$, $\bar{f}=0.07$, $\text{CV}(\tilde{\bar{S}})=0.04$, and the C_i^* were computed to be $C_1^*=0.07$, $C_2^*=0.112$, $C_3^*=0.1372$, and $C_4^*=0.1523$. It is thus easy to compute

$$A'=\frac{1}{C_1^*}+\frac{1}{C_2^*}+\frac{1}{C_3^*}+\frac{1}{C_4^*}=37.06.$$

From Section 9.3, $N=1462$ in this example; thus from formula (9.4.3)

$$M=\frac{\left[\frac{37.06}{0.5}-4\right]}{\left[4\,\text{CV}(\tilde{\bar{S}}')\right]^2-\left[\frac{37.06-4}{1462}\right]},$$

$$M=\frac{70.12}{16\,[\text{CV}(\tilde{\bar{S}}')]^2-0.02261}. \tag{9.4.4}$$

Only now is it necessary to choose a desired value of $\text{CV}(\tilde{\bar{S}}')$. Because $\text{CV}(\tilde{\bar{S}})$ (for adults) was taken as 0.04, one might also want to use this value for young. The value of M is then computed to be

$$M=\frac{70.12}{16(0.04)^2-0.02261}=23{,}452.$$

This is a surprisingly large number, and no doubt impossible to achieve. This points out that it may not be possible to achieve the same coefficient of variation for both adults and young.

The basic problem is that $\text{VAR}(\tilde{\bar{S}}')$ depends upon both the numbers of adults (N) and of young (M) banded. Structurally, $\tilde{S}_i'$ equals $\widetilde{S_i'\rho_{i+1}}/\tilde{\rho}_{i+1}$, where the product $S_i'\rho_{i+1}$ is estimated from band recoveries of young only and ρ_{i+1} is estimated from adult recoveries. Thus the variance of $\tilde{S}_i'$ (and of $\tilde{\bar{S}}'$) has two components, one dependent upon the value of N and one dependent upon the value of M. First-year recoveries from birds banded as young are not used in the estimation of S_i' and this also explains why precision is somewhat poor. Once a value of N is chosen for adults there is a lower bound on the precision of $\tilde{\bar{S}}'$ that can be achieved, even if infinitely many young birds could be banded. In this example with $N=1{,}460$ the lower bound on $\text{CV}(\tilde{\bar{S}}')$ is

$$\text{CV}(\tilde{\bar{S}}')=\sqrt{\frac{0.02261}{N16}}=0.03759\,.$$

In general for N given, the lower bound is

$$\text{CV}(\tilde{\bar{S}}')\geqslant\sqrt{\frac{A'-(k-1)}{N(k-1)^2}}\,, \tag{9.4.5}$$

(which can only be achieved if infinitely many young are banded).

For the example at hand one would realistically have to settle for a lower precision on $\tilde{\bar{S}}'$, for instance $\text{CV}(\tilde{\bar{S}}')=0.07$. Then from formula (9.4.4):

$$M=\frac{70.12}{16(0.07)^2-0.02261}=1{,}257\,.$$

As was pointed out in Section 9.3, an alternative use of these sample size formulae is to determine the precision that will be achieved if a given number of birds is banded (given values of k, $\bar{S}$, etc.) For the case of young, once a value of N is determined, or given, one simply solves formula (9.4.3) for $\text{CV}(\tilde{\bar{S}}')$ given the value of M anticipated.

As a general procedure we recommend computing the unknown terms, like A', to arrive at a reduced formula such as (9.4.4). For this example one then has

$$\mathrm{CV}(\bar{\bar{S}}') = \sqrt{\frac{70.12}{M\,16} + \frac{0.02261}{16}} = \sqrt{\frac{4.3825}{M} + 0.001413} \ .$$

Any number of values of M can now be tried. If it were felt one could band twice as many young as adults, then we have $M = 2(1{,}462) \approx 3{,}000$, and

$$\mathrm{CV}(\bar{\bar{S}}') = \sqrt{\frac{4.3825}{3{,}000} + 0.001413} = 0.0536 \ .$$

Another way to use these formulae for both adults and young is to ask what would be an optimal ratio to band (young/adult) if one wants equal, but unspecified, values of $\mathrm{CV}(\bar{\bar{S}})$ and $\mathrm{CV}(\bar{\bar{S}}')$. The relevant formula is

$$\frac{M}{N} = \frac{\left[\frac{A'}{\bar{\bar{S}}'} - (k-1)\right]}{h(\bar{S}, \bar{f}, k)\ (k-1)^2 - [A' - (k-1)]} \ .$$

Evaluating the formula for the example used throughout these two sections gives

$$\frac{M}{N} = \frac{70.12}{(2.3393)16 - 33.06} = 16.0 \ ,$$

or 16 young per adult. This is likely to be an impossible ratio when reasonable numbers of adults are banded, which fact goes back to our earlier statement that it may be unrealistic to try to achieve the same coefficient of variation for both adults and young.

In summary, the determination of sample sizes (N,M) in case both young and adults are banded is not as simple as determining N alone. It is important to bear in mind the lower limit achievable for $\mathrm{CV}(\bar{\bar{S}}')$ (once a value of N is chosen). This fact may require some adjustments in the target value of N in order to achieve suitable precision for both ages. Even if these formulae are used in no other way, they should be used to determine expected coefficients of variation for $\bar{\bar{S}}$ and $\bar{\bar{S}}'$ in a planned study with prescribed values of N and M. If this precision is too low (coefficients of variation greater than 0.1), the study is nearly worthless for survival rate estimation.