# Small Mammal Population Density with Grid Trapping and Radiotracking

A common problem with trapping small mammals on grids is how to compute density. Density ($D$) is a function of estimated population size ($\hat{N}$) of the grid and the area of the grid ($A$): $\hat{D} = \hat{N}/A$. However, small mammals are typically attracted to the grid, so that the proper value of $A$ is not easily estimated. The approach described here is not used elsewhere in the literature, and involves the use of radios to track the movements of a sample of the animals trapped on the grid. After grid trapping is completed, so that traps no longer affect the movements of the captured animals, the radio-marked individuals are monitored, and the proportion of their locations on the old trapping grid is computed. The trapping grid can be defined as the rectangle formed by the grid, plus maybe a strip around the outside that is ½ the between trap distance on the grid. For each radio-marked animal $j = 1, ..., n$, an estimate of its proportion of time on the grid ($\hat{p}_j$) is the number of locations on the grid divided by the total number of locations taken. The weighted mean of the $\hat{p}_j$ for the radio-marked animals weighted by number of locations taken on each animal provides an adjustment for the population estimate of the grid:

$$\hat{N}_{adj} = \hat{N}\bar{\hat{p}} \ ,$$

where

$$\bar{\hat{p}} = \frac{\sum_{j=1}^{n} w_j \hat{p}_j}{\sum_{j=1}^{n} w_j} \ ,$$

and $w_j$ is the number of locations taken on animal $j$. The variance of $\bar{\hat{p}}$ is computed as

$$\hat{\text{var}}(\bar{\hat{p}}) = \frac{\sum_{j=1}^{n} w_j (\hat{p}_j - \bar{\hat{p}})^2}{(n - 1)\sum_{i=j}^{n} w_j} \ .$$

Thus, the variance of $\hat{N}_{adj}$ can be obtained with the delta method as

$$\hat{\text{var}}(\hat{N}_{adj}) = \hat{N}^2 \hat{\text{var}}(\bar{\hat{p}}) + \bar{\hat{p}}^2 \hat{\text{var}}(\hat{N}) \ ,$$

where $\hat{\text{var}}(\hat{N})$ is obtained from the capture-recapture estimate for the grid trapping captures.

An extension of this approach has been applied to estimation of densities of Preble's Meadow Jumping Mice where density is mice per km of stream (i.e., not mice per area), and mice were only radioed on a few of the grids that were trapped. For the $i = 1, ..., g$ grids where mice were radio-marked, a Michaelis-Menton model was fit to the observed $\bar{\hat{p}}_i$ values (given in Table 1):

$$\bar{\hat{p}}_i = \frac{L_i}{L_i + 2\text{BSW}} + \epsilon_i \ ,$$

where $L_i$ is the length of the trapping line for grid *i*. Nonlinear least squares were used to minimize the sum of squared $\epsilon_i$ values to obtain an estimate of the unknown parameter, BSW (i.e., boundary strip width). Observations were weighted by the number of radio-collared mice providing the observed values of $\bar{\hat{p}}_i$. Because the variance of a proportion is a function of the value of the proportion, a logistic transformation was used to stabilize the variances of the $\epsilon_i$ across the range of the $\bar{\hat{p}}_i$ values:

$$\operatorname{logit}(\bar{\hat{p}}_i) = \ln\left(\frac{\bar{\hat{p}}_i}{1 - \bar{\hat{p}}_i}\right) = \ln\left(\frac{\dfrac{L_i}{L_i + 2\,\mathrm{BSW}}}{1 - \dfrac{L_i}{L_i + 2\,\mathrm{BSW}}}\right) .$$

Table 1. Data on the proportion of locations (*p*) of mouse locations within the trapping grid.

| Site | Year | Session | Trapline Length | No. Mice | Mean *p* Online | SE(*p*) | Weighted Fit | Logistic Weighted Fit |
|---|---|---|---|---|---|---|---|---|
| Walnut Creek A-upper | 1999 | May/June | 325 | 5 | 0.946 | 0.0040 | 0.761 | 0.796 |
| Woodhouse | 1999 | June | 409 | 24 | 0.629 | 0.0843 | 0.800 | 0.831 |
| Woodhouse | 1999 | July | 409 | 19 | 0.902 | 0.0532 | 0.800 | 0.831 |
| Woodhouse | 1999 | Sept | 409 | 6 | 1.000 | 0.0000 | 0.800 | 0.831 |
| Walnut Creek Lower | 1999 | May/June | 490 | 1 | 0.308 | . | 0.827 | 0.855 |
| Maytag | 1998 | Sept | 494 | 16 | 0.954 | 0.0385 | 0.829 | 0.856 |
| Walnut Creek B-series | 1999 | May/June | 500 | 1 | 0.828 | . | 0.830 | 0.858 |
| PineCliff | 1999 | June | 503 | 12 | 0.841 | 0.0670 | 0.831 | 0.858 |
| PineCliff | 1999 | Sept | 503 | 15 | 0.924 | 0.0464 | 0.831 | 0.858 |
| PineCliff | 1999 | July | 503 | 22 | 0.899 | 0.0372 | 0.831 | 0.858 |
| PineCliff | 1998 | July | 504 | 14 | 0.805 | 0.0750 | 0.831 | 0.858 |
| PineCliff | 1998 | Sept | 504 | 16 | 0.671 | 0.0964 | 0.831 | 0.858 |
| Woodhouse | 1998 | July | 516 | 12 | 0.817 | 0.0712 | 0.835 | 0.861 |
| Woodhouse | 1998 | Sept | 574 | 14 | 0.883 | 0.0291 | 0.849 | 0.874 |
| Maytag | 1998 | July | 608 | 16 | 0.719 | 0.0744 | 0.856 | 0.880 |
| Maytag | 1999 | Sept | 1130 | 11 | 0.969 | 0.1664 | 0.917 | 0.932 |
| Maytag | 1999 | July | 1130 | 26 | 0.908 | 0.0364 | 0.917 | 0.932 |
| Maytag | 1999 | June | 1130 | 7 | 0.873 | 0.1339 | 0.917 | 0.932 |

Predicted values of *p* ($\hat{p}$) are obtained from either fitted model for a particular line length (*L*) by substituting the estimate of BSW into the predictive equation:

$$\hat{p} = \frac{L}{L + 2BSW} .$$

Estimates of BSW for the 2 estimation methods are shown in Table 2, predicted values are shown in Table 1, and the fit of the 2 fitted equations is shown in Figure 1.

Table 2. Estimates of boundary strip width (BSW) for nonlinear weighted least squares fit and nonlinear weighted least squares fit with the logistic transformation.

| Method | Estimate of BSW | SE of estimate |
|---|---|---|
| Nonlinear weighted least squares | 51.1241 | 10.1033 |
| Nonlinear weighted least squares with logisitc transformation | 41.5446 | 9.1676 |

Parameter estimates from the logistic transformation model should be used to correct the population estimates, i.e.,

$$\hat{p} = \frac{L}{L + 2\times 41.5446} .$$

This model is preferred because of the variance stabilization provided by the logistic transformation. However, as shown in Figure 1, the difference in the predictions of the 2 fitted equations is not biologically important.

To obtain an adjusted number of mice on a trapping grid, the estimated population on the grid ($\hat{N}$) from mark-recapture estimation is multiplied by the correction ($\hat{p}$) from the above equation to give the adjusted number:

$$\hat{N}_{adj} = \hat{N}\hat{p}.$$

The variance of $\hat{N}_{adj}$ is computed as the variance of a product using the delta method:

$$\hat{\mathrm{var}}(\hat{N}_{adj}) = \hat{N}^2 \hat{\mathrm{var}}(\hat{p}) + \hat{p}^2 \hat{\mathrm{var}}(\hat{N})$$

where

$$\hat{\mathrm{var}}(\hat{p}) = \frac{4L^2 \, \hat{\mathrm{var}}(\mathrm{BSW})}{(L+2\,\mathrm{BSW})^4} .$$

Similarly, density (mice per unit of length of the stream reach) is computed as the adjusted population size divided by the length of the grid, or the original grid population estimate over the adjusted length of the grid:

$$\hat{D} = \frac{\hat{N}_{adj}}{L} = \frac{\hat{N}}{L + 2 * BS\hat{W}} .$$

The variance of $\hat{D}$ is computed using the delta method:

$$\hat{D} = \frac{\hat{\text{var}}(\hat{N}_{adj})}{L^2} = \frac{1}{(L + 2 * B\hat{S}W)^2}\,\hat{\text{var}}(N) + \frac{\hat{N}^2}{(L + 2 * B\hat{S}W)^4}\,\hat{\text{var}}(BSW)\ .$$

Note that $\hat{\text{var}}(\text{BSW}) = \hat{\text{SE}}(\text{BSW})^2$ with SE(BSW) given in Table 2. As an example of the variance of $\hat{p}$, a line length of 500m results in $\hat{\text{SE}}(\hat{p}) = \sqrt{\hat{\text{var}}(\hat{p})}$ = 0.02696. Confidence intervals on the fitted function of $\hat{p}$ are shown in Figure 2, computed as $\hat{p} \pm 1.96 \times \hat{\text{SE}}(\hat{p})$.

An alternative approach to computing confidence intervals on $\hat{p}$ that I would have thought would work better than the $\pm 1.96\hat{\text{SE}}(\hat{p})$ intervals shown in Figure 2 is to compute the confidence interval on the logit($\hat{p}$), and then back-transform the interval endpoints:

$$\frac{1}{1 + \exp\left(\text{logit}(\hat{p}) - 1.96\hat{\text{SE}}(\text{logit}(\hat{p})\right)},\ \frac{1}{1 + \exp\left(\text{logit}(\hat{p}) + 1.96\hat{\text{SE}}(\text{logit}(\hat{p})\right)}$$

where $\hat{\text{SE}}(\text{logit}(\hat{p})) = \dfrac{\hat{\text{SE}}(\text{BSW})}{\text{BSW}}$ = 9.1676/41.5446 = 0.22067 for the data presented here. After computing confidence intervals by both methods, the results were only negligibly different, with the maximum difference in the interval lengths being only 0.00111 over the range of L from 100 to 2000m.

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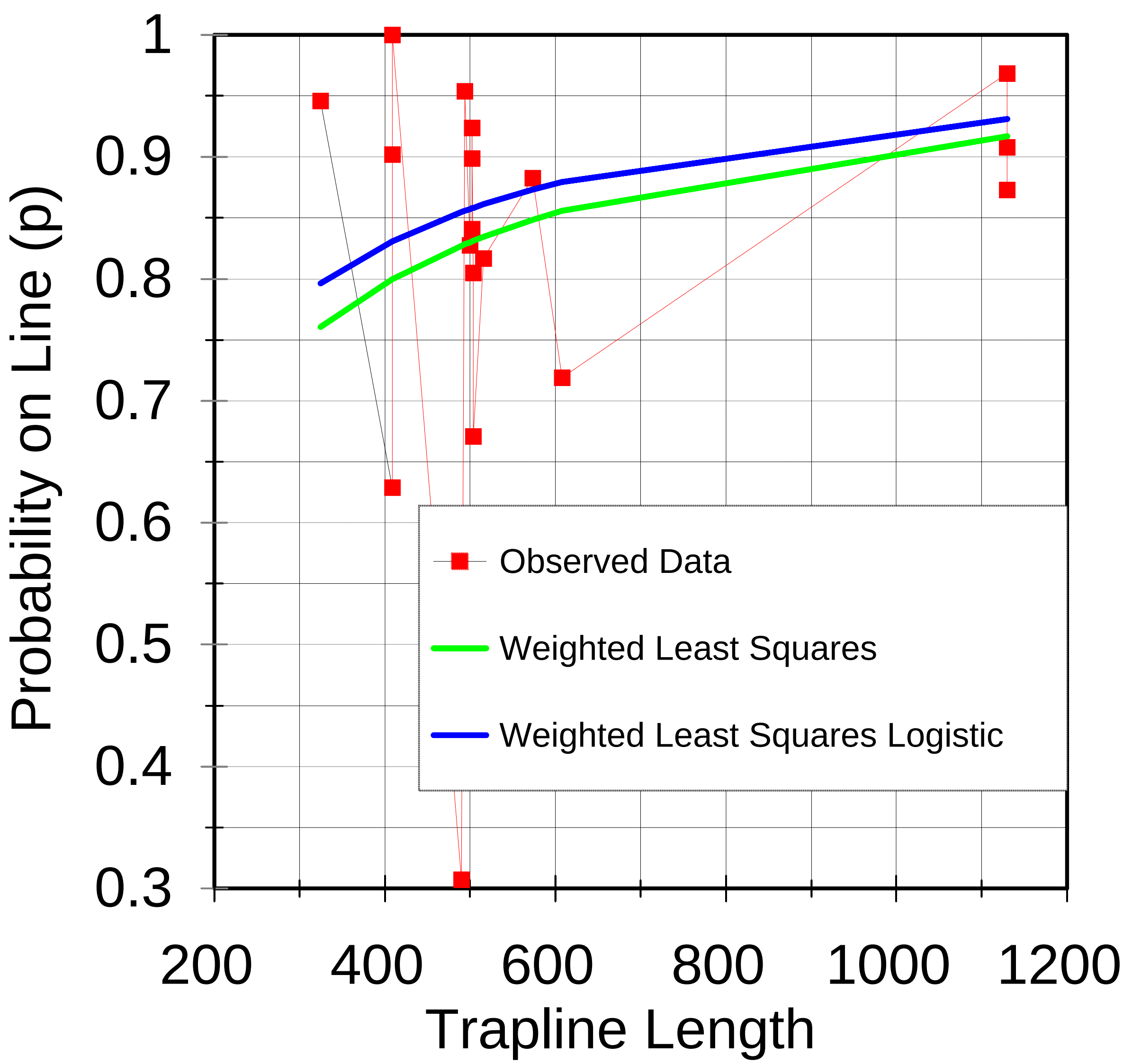


**Figure 1**. Observed estimates of *p* with the fit of the weighted nonlinear least squares model, and the logistic transformed nonlinear least squares model.

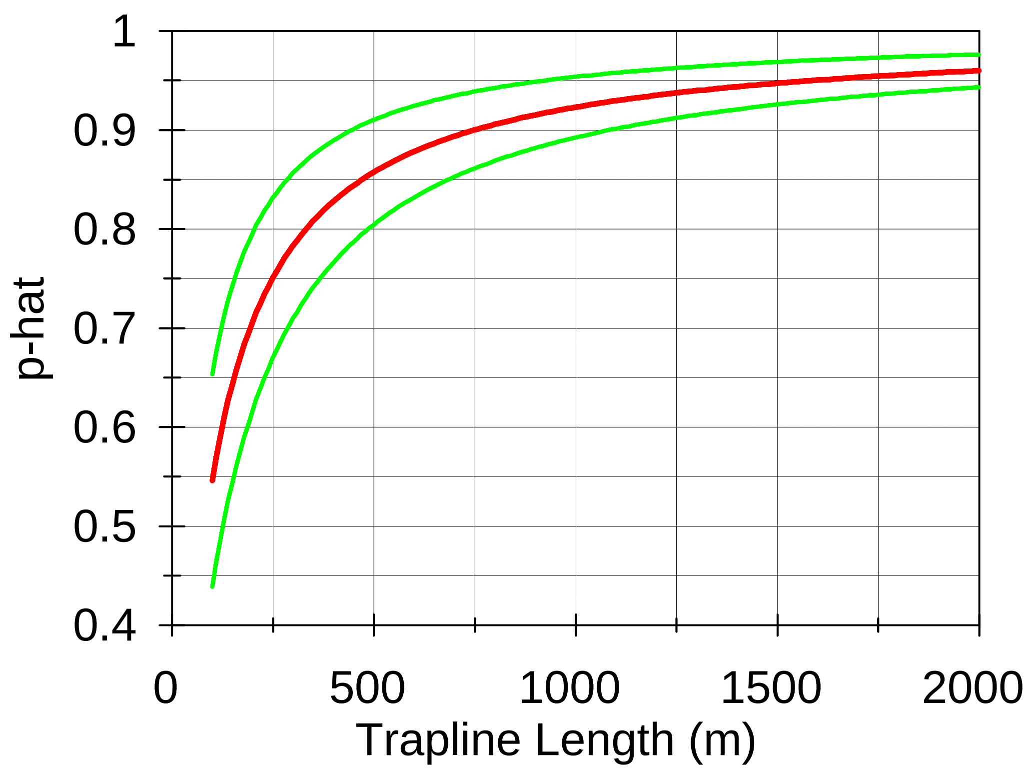


**Figure 2**. Predicted $\hat{p}$ with 95% confidence intervals as a function of trapline length in meters.