Lecture 1. Introduction, Density-independent population growth models; differential vs. difference equations.

Reading:

Gotelli, 2001, A Primer of Ecology, Chapter 1, pages 2-23.

A population is “a group of plants, animals, or other organisms, all of the same species, that live together and reproduce” (Gotelli 1998:2).

What is "population dynamics"? Its an area of investigation between the fields of population biology and population mathematics.

Population Biology	Population Dynamics	Population Mathematics
Little or no concern for mathematical representation	Balance of biology and mathematics	Little or no concern about biological reality

Some questions we would like to be able to answer after this course:

1. Maximum number of adult females that we can harvest sustainably from a mule deer population.
2. Number of individuals needed in a re-introduction, e.g., wolves into Yellowstone, to be fairly certain that the population will persist 100 years.
3. Given data on survival and reproduction, what are the chances that an endangered population will persist 100 years.
4. How large of a marine reserve is required to prevent the extinction of a harvested fish species?

Overview of Course:

4 components of population changes: births, immigration, deaths, emigration (BIDE model)

$N_{t+1} = N_t + B_t + I_t - D_t - E_t$

density dependence/independence,

exponential growth,

logistic growth,

age-structured populations,

stage-structured populations,

additive vs. compensatory mortality,

stochasticity (demographic, environmental, individual heterogeneity, genetic, sampling),

chaos,

metapopulations,

predation,

competition,
herbivory,
evolution, natural selection,
genetics,
social systems,
management of a population,
population viability analysis

Density-independent population growth models for population without age structure

Difference equations for exponential growth

"Birth Pulse" Caughley (1977), see Eberhardt(1992)

Likewise, mortality is not generally constant over the interval, e.g., over-winter mortality of ungulates.

Hence, managers generally think of the population as a operating on a finite interval.

$N_{t+1} = N_t + N_t \times R = N_t \times (1 + R), N_t = N_0 (1 + R)^t$

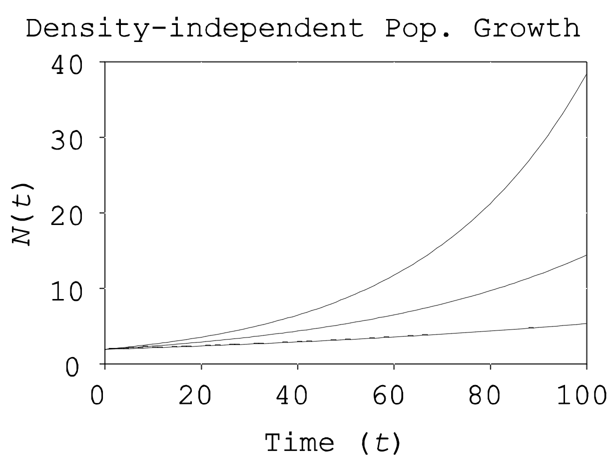
$N_{t+1}/N_t = 1 + R = \lambda$ = annual rate of change:

$\lambda > 1$ increasing, $\lambda < 1$ decreasing

Finite rate of growth (R)

R = finite birth rate - finite death rate + finite immigration rate - finite emigration rate

The following graph is an example of density-independent population growth for $N_0 = 2$, and R = 0.01, 0.02, and 0.03. Note that I sometimes use $N(t)$ in place of N_t in labels on graphs.



Differential equations for exponential growth

Continuous births, deaths, immigration, emigration

$$\frac{dN}{dt} = rN$$

With integration, $N_t = N_0 e^{rt}$ where $e^r = \lambda = N_{t+1}/N_t$, and hence the name exponential growth for this model.

$$e^r = 1 + r + \frac{r^2}{2!} + \frac{r^3}{3!} + \ldots$$

$$\log(1 + r) = r - \frac{r^2}{2} + \frac{r^3}{3} - \frac{r^4}{4} + \ldots$$

Instantaneous rate of growth (r)

r = instantaneous birth rate - instantaneous death rate + instantaneous immigration rate - instantaneous emigration rate

Continuous time and instantaneous birth rate is not appropriate for a population with a birth pulse.

Distinction between differential and difference (continuous vs. discrete) becomes important mathematically only when r is reasonably large, $\geq$0.2 or 0.3, where we have to choose between $1 + r = 1.2$ and $e^r = 1.22$, or $1 + r = 1.3$ and $e^r = 1.35$ (Eberhardt 1992, Journal of Wildlife Management 56:603-610).

Conversion between R and r, $1 + R = e^r$

In the following table, let $N_0 = 10$, $R = r = 0.2$.

t	$N_t = N_0(1 + R)^t$	$N_t = N_0 e^{rt}$
0	10	10
1	12	12.2140275816
2	14.4	14.9182469764
3	17.28	18.2211880039
4	20.736	22.2554092849
5	24.8832	27.1828182846
6	29.85984	33.2011692274

Distinction between r and R is more important biologically, i.e., pulse reproduction versus continuous reproduction, instantaneous vs. finite survival rates.

Confusion in literature of notation, r, R, λ. Be careful to understand the author's definitions of notation

Exponential growth = density independence

Density independence, *R* and *r* are not functions of N_t, i.e., births, deaths, immigration, and emigration are not functions of N_t. In other words, no matter what the population size, the per capita birth rate and per capita death rate are the same as for all other population sizes.

Programming of difference equations.

An example of how spreadsheet equations are iterative on the previous value. Time is in the A column, and the population size is in the B column. Row 1 initialize time to 0 and the population to 100. Row 2 represents the population at time 1, row 3 at time 2, etc.

Row/Col	A	B
1	0	100 (=N_0)
2	+A1+1	+B1*(1+*R*)
3	+A2+1	+B2*(1+*R*)
4	+A3+1	+B3*(1+*R*)
5	+A4+1	+B4*(1+*R*)
6	+A5+1	+B5*(1+*R*)
7	+A6+1	+B6*(1+*R*)

Laboratory Exercise 1: Difference equation model of exponential growth in Quattro Pro.

Literature Cited

Caughley, G. 1977. Analysis of vertebrate populations. Wiley & Sons, New York, New York, USA.234 pp.

Eberhardt, L. L. 1992. Assessing rates of increase from trend data. Journal of Wildlife Management 56:603-610.