

Lecture 3. Effects of time delays, overcompensation, chaos, on logistic-type models.

Reading:

Renshaw (1991) Chapter 4, Time-lag models of population growth, pages 87-127, and

Optional:

Ritchie, M. E. 1992. Chaotic dynamics in food-limited populations: implications for wildlife management. Pages 139-147 in D. R. McCullough and R. H. Barrett, eds. *Wildlife 2001: Populations*. Elsevier Applied Science, New York, New York

Impact of environmental variation on K , R_0 , resulting N_{t+1}

K is a random variable

Lots of variance in K appears to be random fluctuations because the population is always approaching K

Less variation gives a population that fluctuates around K

Examples: overwinter survival in Piceance mule deer fawns

R_0 gives a random approach towards K

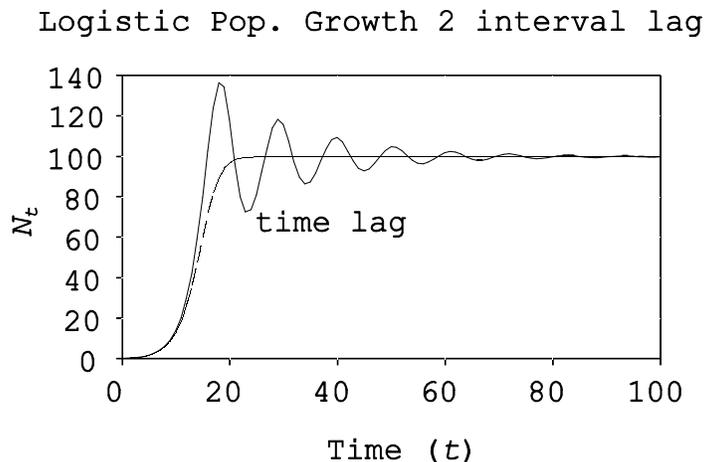
Examples: water temperature, acidity of amphibian reproduction, number of ponds for North American mallards.

Biological reasons for time delays.

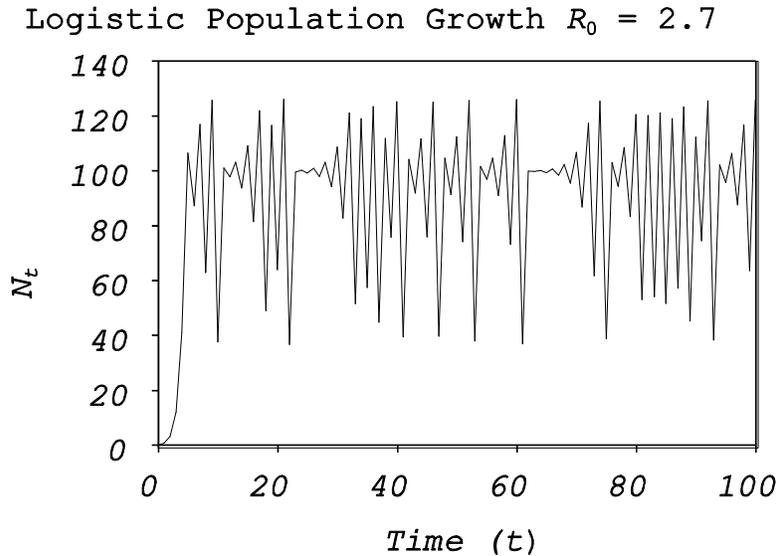
$$N_{t+1} = N_t \left[1 + R_0 \left(1 - \frac{N_{t-\tau}}{K} \right) \right]$$

17-year locusts, salmon, length of parturition > 1 year?

Behavior of difference equation logistic model with time delays -- cycles. In the following graph, a population with density-dependence based on population size 2 times ago is plotted on top of the expected logistic function with no time lag.



Chaos. When R_0 exceeds 2, the population behaves erratically.



Dynamical Behavior	Value of R_0
Stable equilibrium point K	$0 < R_0 < 2$
Stable 2-point cycle	$2 < R_0 < 2.5$
Stable 4-point cycle	$2.5 < R_0 < 2.55$
Stable cycles, period 8, then 16 then 32, etc.	$2.55 < R_0 < 2.57$
Chaos	$2.57 < R_0$

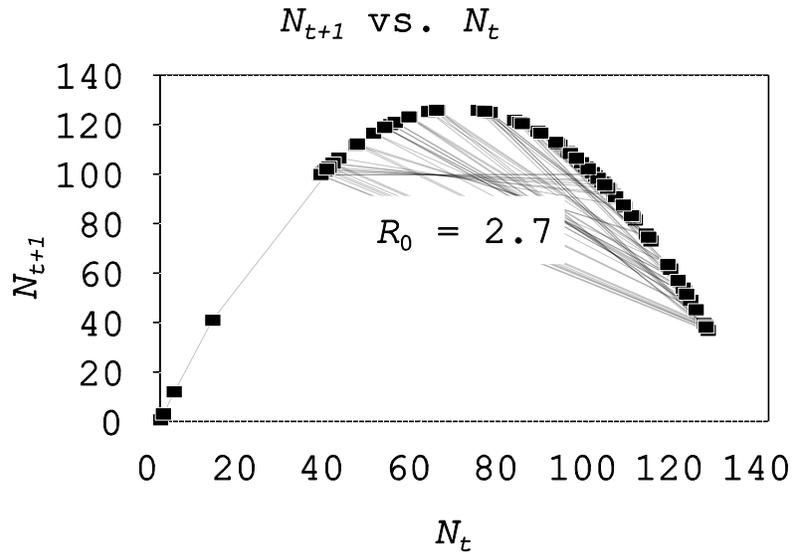
Taken from Renshaw (1991:101).

Biological arguments for chaos: populations behave erratically when observed.

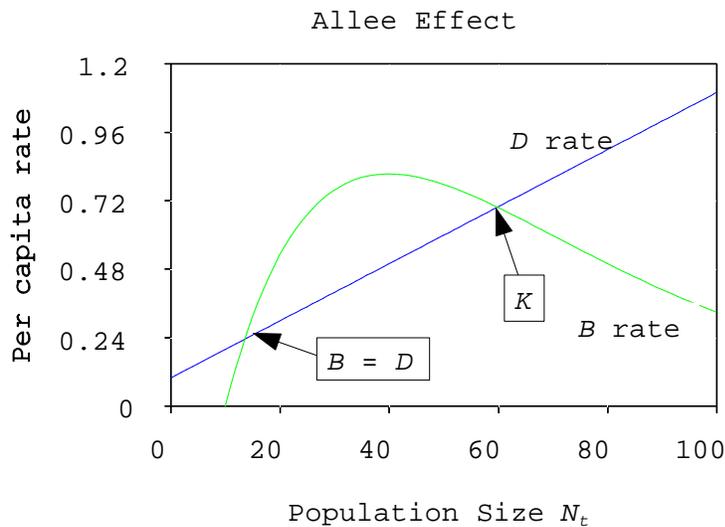
They seldom, if ever, stabilize at a single value.

Mathematical reasons for chaos, vs. biological reality: Chaos implies that

population growth rate exceeds the rate at which density-dependence feeds back into the process, i.e., the population overcompensates for the existing density by either growing too fast or declining too fast and overshooting K . A value of $R_0 = 2.6$ implies that the population can increase 2.6X before any density-dependent responses take place to dampen $R(t)$. Further, even though there appears to be no pattern in the time trace of the population, a pattern is present. All the consecutive pairs of population sizes fall on a parabola!



Allee effect is that the per capita birth rate declines at low densities because, for example, of the increased difficulty of finding a mate (Yodzis 1989:12-13). This is known as *Allee-type behavior* (of the per capita birth rate), and its effect on the growth rate $R(t)$ is called an *Allee effect* (Allee 1931).



Literature Cited

Allee, W. C. 1934. Animal aggregations: A study in general sociology. University Chicago Press, Chicago.

Renshaw, E. 1991. Modelling biological populations in space and time. Cambridge University Press, New York, NY. 350 pp.

Yodzis, P. 1989. Introduction to theoretical ecology. Harper and Row, New York, New York, USA. 383 pp.