Lecture 3. Effects of time delays, overcompensation, chaos, on logistic-type models.

Reading:
Renshaw (1991) Chapter 4, Time-lag models of population growth, pages 87-127, and

Optional:

Impact of environmental variation on \( K, R_0 \), resulting \( N_{t+1} \)

- \( K \) is a random variable
  - Lots of variance in \( K \) appears to be random fluctuations because the population is always approaching \( K \)
  - Less variation gives a population that fluctuates around \( K \)
  - Examples: overwinter survival in Piceance mule deer fawns
  - \( R_0 \) gives a random approach towards \( K \)
    - Examples: water temperature, acidity of amphibian reproduction, number of ponds for North American mallards.

Biological reasons for time delays.

\[
N_{t+1} = N_t \left[ 1 + R_0 \left( 1 - \frac{N_{t-\tau}}{K} \right) \right]
\]

17-year locusts, salmon, length of parturition > 1 year?

Behavior of difference equation logistic model with time delays -- cycles. In the following graph, a population with density-dependence based on population size 2 times ago is plotted on top of the expected logistic function with no time lag.

![Logistic Pop. Growth 2 interval lag](image-url)
Chaos. When $R_0$ exceeds 2, the population behaves erratically.

**Logistic Population Growth $R_0 = 2.7$**

<table>
<thead>
<tr>
<th>Dynamical Behavior</th>
<th>Value of $R_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable equilibrium point $K$</td>
<td>$0 &lt; R_0 &lt; 2$</td>
</tr>
<tr>
<td>Stable 2-point cycle</td>
<td>$2 &lt; R_0 &lt; 2.5$</td>
</tr>
<tr>
<td>Stable 4-point cycle</td>
<td>$2.5 &lt; R_0 &lt; 2.55$</td>
</tr>
<tr>
<td>Stable cycles, period 8, then 16 then 32, etc.</td>
<td>$2.55 &lt; R_0 &lt; 2.57$</td>
</tr>
<tr>
<td>Chaos</td>
<td>$2.57 &lt; R_0$</td>
</tr>
</tbody>
</table>

Taken from Renshaw (1991:101).

Biological arguments for chaos: populations behave erratically when observed. They seldom, if ever, stabilize at a single value.

Mathematical reasons for chaos, vs. biological reality: Chaos implies that population growth rate exceeds the rate at which density-dependence feeds back into the process, i.e., the population overcompensates for the existing density by either growing too fast or declining too fast and overshooting $K$. A value of $R_0 = 2.6$ implies that the population can increase 2.6X before any density-dependent responses take place to dampen $R(t)$. Further, even though there appears to be no pattern in the time trace of the population, a pattern is present. All the consecutive pairs of population sizes fall on a parabola!
Allee effect is that the per capita birth rate declines at low densities because, for example, of the increased difficulty of finding a mate (Yodzis 1989:12-13). This is known as Allee-type behavior (of the per capita birth rate), and its effect on the growth rate $R(t)$ is called an Allee effect (Allee 1931).
Literature Cited

