FW663 -- Laboratory Exercise

Computer Algebra and Methods for Estimating Variances

Part 1. DERIVE for Symbolic Algebra

In this exercise, we will introduce a computer package called DERIVE that performs most of the symbolic math needed for this course. DERIVE is handy for people whose mathematical skills are rusty. Other symbolic algebra packages include Maple and Macsyma. Only DERIVE is available on the CNR network, but Maple is available on Lamar.

We will demonstrate the use of DERIVE in class to get you started with the package. Using DERIVE, solve the following problems.

1. Solve for N in the following expression:

$$\left(1 - \frac{M_{t+1}}{N}\right) = \left(1 - \frac{n_1}{N}\right)\left(1 - \frac{n_2}{N}\right)$$

2. Substitute the expression $M_{t+1} = n_1 + n_2 - m_2$ into the above solution and simplify the result.

3. Differentiate the following multinomial likelihood and set the result equal zero to obtain estimators of p_1 and p_2:

$$\binom{N}{n_1 \; n_2} p_1^{n_1} p_2^{n_2} (1 - p_1 - p_2)^{(N - n_1 - n_2)}$$

You may want to take the log first. The correct estimators are $\hat{p}_1 = n_1/N$ and $\hat{p}_2 = n_2/N$.

4. Solve for the estimators of the variance of p_1 and p_2, and the covariance of p_1 and p_2. Remember, you have to take the log of the likelihodd before taking second partials. The correct estimators are

$$\hat{\text{var}}(\hat{p}_1) = \frac{\hat{p}_1 \ (1 - \hat{p}_1)}{N} \ , \ \hat{\text{var}}(\hat{p}_2) = \frac{\hat{p}_2 \ (1 - \hat{p}_2)}{N} \ \text{, and}$$

$$\hat{\text{cov}}(\hat{p}_1, \ \hat{p}_2) = \frac{-\hat{p}_1 \hat{p}_2}{N} \ .$$

5. Extra challenge for the serious student because DERIVE doesn't do well with this problem. Solve for N in the following expression (try modifying the previous expression to add the 3rd term):

$$\left(1 - \frac{M_{t+1}}{N}\right) = \left(1 - \frac{n_1}{N}\right)\left(1 - \frac{n_2}{N}\right)\left(1 - \frac{n_3}{N}\right)$$

Some tricks to remember:

1) change the format of input in DERIVE to words before authoring any expressions,

2) break the multinomial coefficient in problem 3 down into factorials (DERIVE uses the ! sign for factorial), or use the COMB(n,y) function (COMB stands for combinations),

3) DERIVE uses the ^ sign for exponentiation,

4) construct a matrix as [[a,b], [c,d]],

5) invert a matrix as 1/matrix or matrix^{-1},

6) retrieve a previous expression as #10 (retrieves expression 10),

7) the function to compute the natural logarithm to the base e of x is LN(x), i.e., LN in caps, and

8) use the arrow keys and the F3 key to retrieve portions of previous expressions.

Part 2. Delta Method

The problem of computing the variance of an estimate that is constructed from other estimates is common The delta method is an analytical approach that requires you to specify the derivatives of the function with respect to the random variables used as input. Hence, DERIVE can be useful to obtain these derivatives.

Use the delta method (and possibly DERIVE) to compute the variance of the following estimates.

1. Mean Life Span (MLS), computed from an estimate of survival as MLS = -1/log($\hat{S}$).

2. The recovery rate, $\hat{f}_i$, computed as $\hat{f}_i = (1 - \hat{S}_i)\hat{r}_i$. Don't assume that the sampling covariance of $\hat{S}_i$ and $\hat{r}_i$ are equal to zero.

3. Rate of population increase, $\hat{\lambda}$, defined as

$$\hat{\lambda} = [\hat{s} + (\hat{s}^2 + 4\hat{s}_0\hat{s}_1\hat{b})^{1/2}]/2$$

   where $\hat{s}$ is estimated adult survival, $\hat{s}_0$ is estimated juvenile survival, $\hat{s}_1$ is estimated subadult survival, and $\hat{b}$ is estimated fecundity (Noon and Biles 1990). Don't assume that $\hat{s}$, $\hat{s}_0$, and $\hat{s}_1$ have zero covariance, but that $\hat{b}$ does have zero covariance with the 3 survival rate estimates.

Part 3. Bootstrap Methods

The bootstrap procedure is a numerical approach to estimating the variance of an estimate that is constructed from data. As an example, suppose that you want to compute the variance of the MLS from a sample of radio-collared animals. To do this, you would draw bootstrap samples from the original sample of animal fates, compute a survival estimate, and then compute MLS. I have provided the SAS code in the file

J:\CLASSES\FW663\EXERCISE.21\BOOT_MLS.SAS

to illustrate the computation of the variance of a binomial estimate of survival and MLS with the bootstrap procedure. Fates of animals, coded as 0=died and 1=lived, are included in the file.

For your information, the file

J:\CLASSES\FW663\EXERCISE.21\DATAGEN.SAS

contains the code used to generate the data in the previous 2 examples. I used a beta-binomial distribution to provide heterogeneity.

Questions for Discussion

1. What example do you have from your research that needs the application of the delta or bootstrap methods?

2. How close does the bootstrap estimate of variance from the SAS code come to the delta method estimate from the original sample?

3. How come the MLS computed from the mean of the bootstrap values of $\hat{S}$ doesn't equal the mean of the MLS bootstrap values? Hint: look up Jensen's Inequality in a mathematical statistics book.

Literature Cited

Noon, B. R., and C. M. Biles. 1990. Mathematical demography of spotted owls in the Pacific Northwest. Journal of Wildlife Management 54:18-27.